12.9B

15. $\frac{d}{dx} f(x) = \frac{-1}{5-x} = \frac{-1/5}{1-x/5} = \sum_{n=0}^{\infty} -\frac{1}{5}\left(\frac{x}{5}\right)^n = -\sum_{n=0}^{\infty} \frac{1}{5^{n+1}} x^n$, $\left|\frac{x}{5}\right| < 1$, $|x| < 5$

$$f(x) = -\int \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} x^n = C + -\sum_{n=0}^{\infty} \frac{1}{5^{n+1}} \cdot \frac{x^{n+1}}{n+1} = \ln(5-x)$$

let $x = 0$

$$C - 0 = \ln(5-0)$$

$$C = \ln 5$$

$$f(x) = \ln 5 - \sum_{n=0}^{\infty} \frac{1}{5^{n+1}} \frac{x^{n+1}}{n+1} \text{ or } \ln 5 - \sum_{n=1}^{\infty} \frac{1}{5^n} \cdot \frac{x^n}{n}, \text{ radius} = 5$$

16) $f(x) = x^2 \left[\overset{g(x)}{\frac{1}{(1-2x)^2}}\right]$

$\int g(x) = \frac{1/2}{1-2x} = \sum_{n=0}^{\infty} \frac{1}{2}(2x)^n = \sum_{n=0}^{\infty} 2^{n-1} x^n$, $|2x| < 1$, $|x| < \frac{1}{2}$

$f(x) = x^2 \sum_{n=1}^{\infty} 2^{n-1} n x^{n-1}$

$g(x) = \sum_{n=1}^{\infty} 2^{n-1} n x^{n-1}$

$$f(x) = \sum_{n=1}^{\infty} 2^{n-1} n x^{n+1} \text{ or } \sum_{n=2}^{\infty} 2^{n-2}(n-1) x^n, \text{ radius} = \frac{1}{2}$$

17) $f(x) = x^3 \left[\overset{g(x)}{\frac{1}{(x-2)^2}}\right]$

$\int g(x) = \frac{-1}{x-2} = \frac{1}{2-x} = \frac{1/2}{1-x/2} = \sum_{n=0}^{\infty} \frac{1}{2}\left(\frac{x}{2}\right)^n$, $\left|\frac{x}{2}\right| < 1$

$g(x) = \frac{d}{dx} \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n+1} n x^{n-1}$

$$f(x) = x^3 \sum_{n=1}^{\infty} \frac{n}{2^{n+1}} x^{n-1} = \sum_{n=1}^{\infty} \frac{n}{2^{n+1}} x^{n+2} \text{ or } \sum_{n=3} \frac{n-2}{2^{n-1}} x^n, \text{ radius} = 2$$

18) $f(x) = \tan^{-1}\left(\frac{x}{3}\right)$

$\frac{d}{dx}\tan^{-1}\left(\frac{x}{3}\right) = \frac{1}{1+\left(\frac{x}{3}\right)^2}\cdot\frac{1}{3}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{3^{2n+1}(2n+1)}$

$\tan^{-1}\left(\frac{x}{3}\right) = \int \sum_{n=0} \frac{1}{3}\left(-\frac{x^2}{3^2}\right)^n = \frac{(-1)^n x^{2n}}{3^{2n+1}}$

$= C + \sum_{n=0} \frac{(-1)^n x^{2n+1}}{3^{2n+1}(2n+1)}$

$\left|\frac{x}{3}\right| < 1$

$|x| < 3$

$R = 3$

let $x = 0$

$\tan^{-1}\left(\frac{0}{x}\right) = C + 0$

$0 = C$

19) $f(x) = \ln(3+x)$

$\frac{d}{dx}\ln(3+x) = \frac{1}{3+x} = \frac{1/3}{1-(-x/3)}$

$\left|\frac{-x}{3}\right| < 1$ $f(x) = \ln 3 + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{3^{n+1}(n+1)}$

$\ln(3+x) = \int \sum_{n=0}^{\infty} \frac{1}{3}\left(-\frac{x}{3}\right)^n = \int \sum \frac{(-1)^n x^n}{3^{n+1}}$

$|x| < 3$ $f(x) = \ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{3^n n}$

$R = 3$

$= C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{3^{n+1}(n+1)}$

let $x = 0$

$\ln(3) = C + 0$

20) $f(x) = \frac{1}{x^2+25} = \frac{1/25}{1-(-x^2/25)}$

$= \sum_{n=0}^{\infty} \frac{1}{25}\left(-\frac{x^2}{25}\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{25^{n+1}}$

$\left|\frac{-x^2}{25}\right| < 1$

$|x^2| < 25$

$|x| < 5$

$R = 5$

$\frac{d}{dx}\ln(1+x) = \frac{1}{1+x} = \sum$

$\ln(1+x) = \int \sum (-x)^n$

$\ln(1+x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

$C = \ln 1 = 0$

$\frac{d}{dx}\ln(1-x) = \frac{1}{1-x}$

$\ln(1-x) = \int \sum_{n=0} x^n$

$= \sum \frac{x^{n+1}}{n+1}$

21) $f(x) = \ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$

$|x| < 1$

$\sum^{\infty} (-1)^n x^{n+1} \quad \sum^{\infty} x^{n+1}$

22) $f(x) = \tan^{-1}(2x)$ $\qquad \frac{d}{dx}\tan^{-1}(2x) = \frac{1}{1+4x^2}\cdot 2$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1} x^{2n+1}}{2n+1}$$

$$\tan^{-1}(2x) = \int \frac{2}{1-(-4x^2)}$$

$$= \int \sum_{n=0} 2(-4x^2)^n$$

$$= \int \sum_{n=0} 2(-4)^n x^{2n}$$

$$= C + \sum_{n=0} \frac{2(-4)^n x^{2n+1}}{2n+1}$$

let $x=0$

$\tan^{-1}(2x) = C+0$

$0 = C$

$|-4x^2| < 1$

$|x^2| < \frac{1}{4}$

$|x| < 1/2$

$R = 1/2$ $\qquad$ endpts converge AST

$[-1/2, 1/2]$ $\qquad \sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}, \sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

23) $\int \frac{t}{1-t^8}\,dt = \int t \cdot \frac{1}{1-t^8}\,dt = \int t \sum_{n=0}^{\infty} (t^8)^n$

$$= \int \sum_{n=0}^{\infty} t^{8n+1}$$

$$= C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}$$

$|t^8| < 1$

$|t| < 1$

$R = 1$

24) $\int \frac{1}{t} \cdot \ln(1-t)\,dt$

$$= \int \frac{1}{t} \cdot \sum_{n=0}^{\infty} \frac{(-1)t^{n+1}}{n+1}\,dt$$

$$= \int \sum_{n=0} \frac{(-1)\,t^n}{n+1}\,dt$$

$$= C - \sum_{n=0}^{\infty} \frac{t^{n+1}}{(n+1)^2}$$ radius $= 1$

$\frac{d}{dx}\ln(1-t) = \frac{1}{1-t} \cdot -1 = \sum_{n=0}^{\infty} (-1)t^n$

$\ln(1-t) = \int \sum_{n=0}^{\infty} (-1) t^n$

$\ln(1-t) = C + \sum_{n=0}^{\infty} \frac{(-1)t^{n+1}}{n+1}$

let $t=0$

$0 = C+0$

25) $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=} (-1)^n x^{2n}$

$\tan^{-1}x = \int \sum_{n=0} (-x^2)^n = \int \sum (-1)^n x^{2n}$

$= C + \sum_{n=0} \frac{(-1)^n x^{2n+1}}{2n+1}$

$x - \tan^{-1}x = \cancel{x} - \left(\cancel{\frac{x}{1}} - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\right)$

$= \sum_{n=1} \frac{(-1)^{n+1} x^{2n+1}}{2n+1}$

$\frac{x - \tan^{-1}x}{x^3} = \frac{1}{x^3}\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-2}}{2n+1}$

$\int \frac{(-1)^n x^{2n-2}}{2n+1} = C + \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n+1)(2n-1)}$

$4x^2-1$