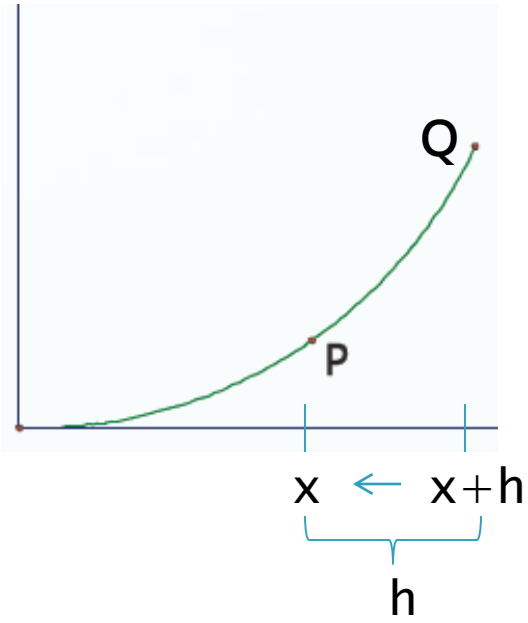
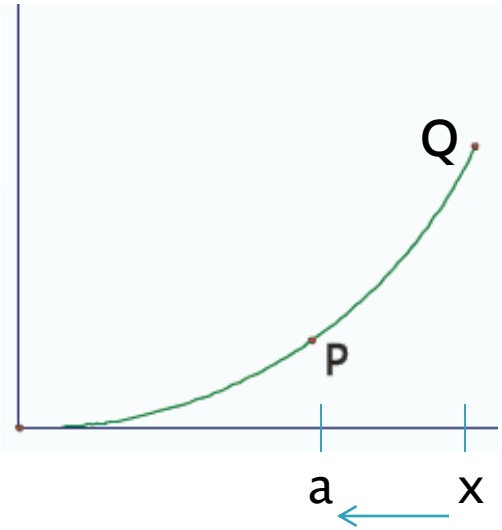
Chapter 3 Derivatives

Second Definition



Definition #1: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$

Distance between points is approaching zero



P(a, f(a)) Q (x, f(x))

Point Q is approaching Point P

Definition #2: $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

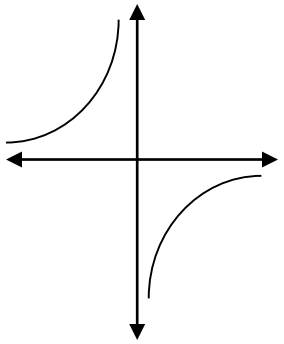
$f(x) = 4 - x^2$ Find f'(1)

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

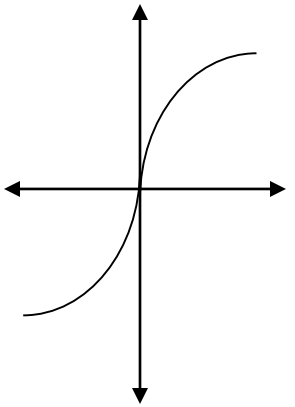
- If a function is differentiable (adjective) at a point, you can differentiate (verb) to find the derivative (noun).
- To be differentiable at a point, the function must be continuous at that point.
- But being continuous at a point does not guarantee a function is differentiable at that point.

When derivatives fail to exist

1. Discontinuous at a point
   If f(0) is undefined, then f'(0) DNE



2. Vertical tangent at a point
   Slope of a vertical line is undefined
   so the derivative does not exist



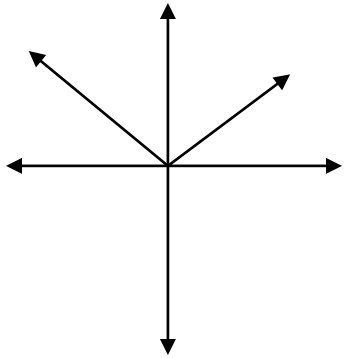
When derivatives fail to exist

- 3. Corner

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = 1$$

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = -1$$

Limits(slopes) do not agree at x = 0, so derivative does not exist

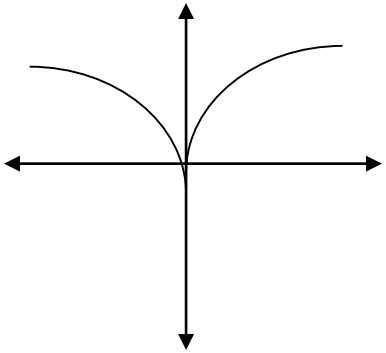


- 4. Cusp

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = -\infty$$

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = +\infty$$

Limits(slopes) do not agree at x = 0
And tangent is vertical, so slope is undefined



(Zoom in–does it look like a line?)

Estimating derivatives of a function defined by a table of values

- Example: Let D(t) be the U.S. National debt at time t. D(t) is measured in billions of dollars.
- Estimate and interpret D'(1990)

t	D(t)
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2

D’(1990) can be calculated two ways.

1. “straddle the point”

Calculate the rate of change from 1985 to 1995.

t	D(t)
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2

D'(1990) can be calculated two ways.

2. Calculate the rate of change on both sides, then average the two rates.

t	D(t)
1980	930.2
1985	1945.9
1990	3233.3
1995	4974.0
2000	5674.2

1985 to 1990

1990 to 1995

D'(1990) = approx 303 billion dollars per yr.
This is the rate the debt was increasing in 1990.