Sec 3.6 Chain Rule

Problem: Find the derivative of $f(x) = (2x^2 - 4x + 1)^{60}$

Chain Rule: $\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$

$f(x) = (2x^2 - 4x + 1)^{60}$

$f(x) = [g(x)]^{60}$

$g(x) = 2x^2 - 4x + 1$

$f'(x) = 60(\underline{2x^2 - 4x + 1})^{59} \cdot (4x - 4)$

ex) $f(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$

outside $(\ )^{1/2}$ inside x^2+1

$$f'(x) = \tfrac{1}{2}(\underline{x^2+1})^{-1/2} \cdot 2x$$
$$= \frac{x}{\sqrt{x^2+1}}$$

ex) $f(x) = \sin^2 x = [\sin x]^2$

outside $[\ ]^2$ inside $\sin x$

$$f'(x) = 2[\underline{\sin x}]^1 \cdot \cos x$$
$$= 2\sin x \cdot \cos x$$

You Try:

$$y = (3x-4)^{10}$$
$$y' = 10(3x-4)^9 \cdot 3$$
$$= 30(3x-4)^9$$

$$y = \sin(7-5x)$$
$$y' = \cos(7-5x) \cdot -5$$
$$= -5\cos(7-5x)$$

Chain Rule with other rules

$$y = (2x+1)^5(x^3-x+1)^4$$

$$y' = (2x+1)^5 \cdot 4(x^3-x+1)^3 \cdot (3x^2-1) + 5(2x+1)^4 \cdot 2 \cdot (x^3-x+1)$$

$$= 4(2x+1)^5(x^3-x+1)^3(3x^2-1) + 10(2x+1)^4(x^3-x+1)^4$$

$$g(t) = \left(\frac{t-2}{2t+1}\right)^9$$

$$\overset{5}{\cancel{2t}+1-\cancel{2t}+4}$$

$$g'(t) = 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \left(\frac{(2t+1)(1)-(t-2)2}{(2t+1)^2}\right)$$

$$= \frac{45(t-2)^8}{(2t+1)^{10}}$$

Using Chain Rule Multiple Times

ex) $y = (1 + \cos(2x))^2$

$$y' = 2(\underline{1 + \cos(2x)})^1 \cdot (0 + {}^-\sin(\underline{2x}) \cdot 2$$

$$= -4\sin(2x)[1 + \cos(2x)]$$

ex) $y = \tan[\underline{5 - \sin(2t)}]$

$$y' = \sec^2[\underline{5 - \sin(2t)}] \cdot [0 - \cos(2t)] \cdot 2$$

$$= -2\cos(2t)[\sec^2(5 - \sin(2t))]$$

ex) $y = \sqrt{\sec(x^3)} = [\sec(x^3)]^{1/2}$

$$y' = \tfrac{1}{2}\left[\underline{\sec(x^3)}\right]^{-1/2} \cdot \sec(x^3)\tan(x^3) \cdot 3x^2$$

$$= \frac{3x^2 \sec(x^3)\tan(x^3)}{2\sqrt{\sec(x^3)}}$$