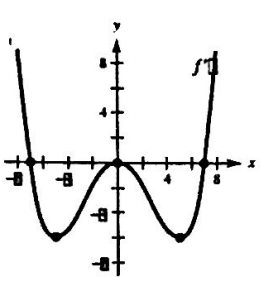
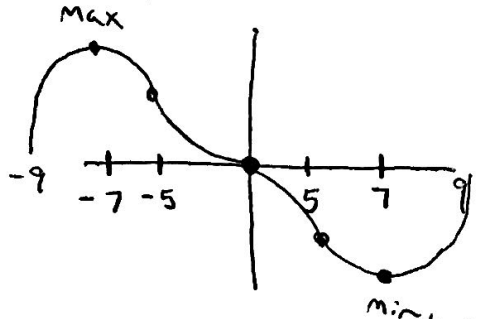
## Connecting Derivatives to Graph of Functions

1. Consider the graph of $f'$, the derivative of $y = f(x)$ defined on the domain $-9 < x < 9$.




(a) For what values of $x$ does $f$ have a relative minimum? at $x = 7$

(b) For what values of $x$ does $f$ have a relative maximum? at $x = -7$

(c) Determine the open intervals where the graph of $f$ is concave downwards. Show the analysis that leads to your conclusion. $(-9, -5)$ $(0, 5)$ b/c $f'$ is decreasing

(d) Sketch the graph of $f$ on the interval $(-9, 9)$ if $f(0) = 0$. Show the analysis that leads to your graph.

Dec $(-9, 0)(0, 7)$

2. Given the graph of $f'(x)$ to the right,

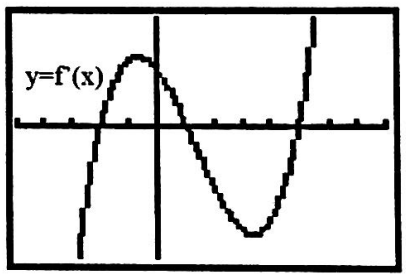
a) where is $f(x)$ increasing? $(-2, 1)$ $(5, \infty)$

b) where is $f(x)$ decreasing? $(-\infty, -2)$ $(1, 5)$

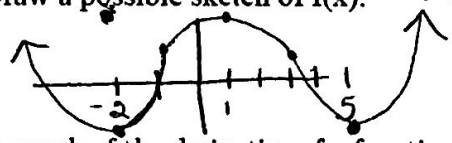
c) identify all critical points of $f(x)$. $x = -2, 1, 5$

d) identify where all relative maximum and minimum values occur.

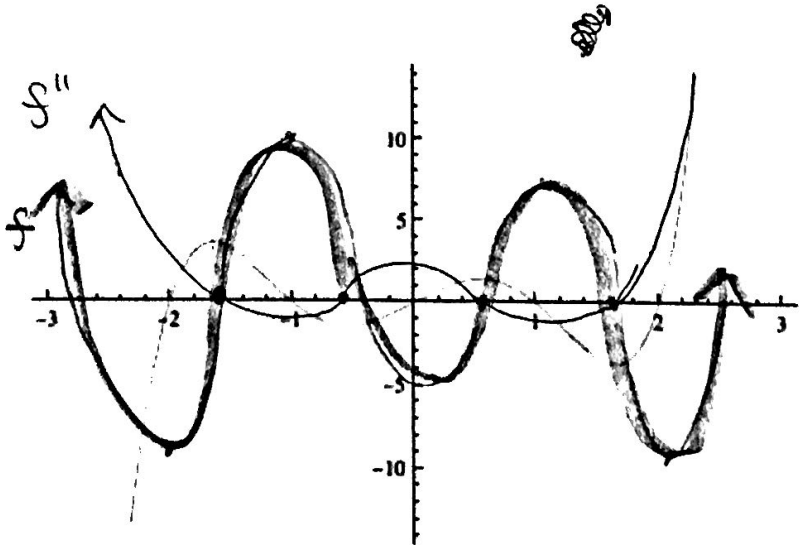
e) Draw a possible sketch of f(x).




min $x = -2$ max $x = 1$
$x = 5$




3. The graph of the derivative of a function (*i.e.* $f'(x)$) is sketched below. Sketch its derivative $f''(x)$, and then use both of those graphs to make a possible sketch of the original function, $f(x)$. Pay attention to the concavity of $f$ as well as where it is increasing and decreasing.

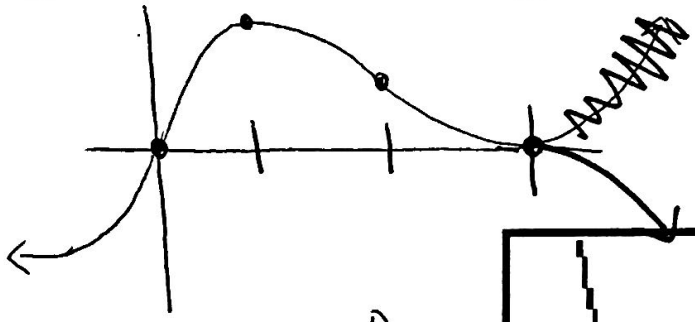


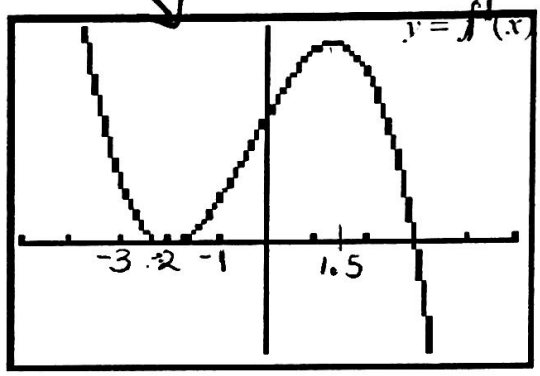
## Connecting Derivatives to Graph of Functions

4. The characteristics of a function are given in the chart below. Sketch a possible graph of the function.

Max

| | $x<0$ | $x=0$ | $0<x<1$ | $x=1$ | $1<x<2$ | $x=2$ | $2<x<3$ | $x=3$ | $x>3$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | - | 0 | + | + | + | + | + | 0 | - |
| $f'(x)$ | + | + | + | 0 | - | - | - | 0 | - |
| $f''(x)$ | + | 0 | - | - | - | 0 | + | 0 | - |
| | | IP | | | | IP | | IP | |





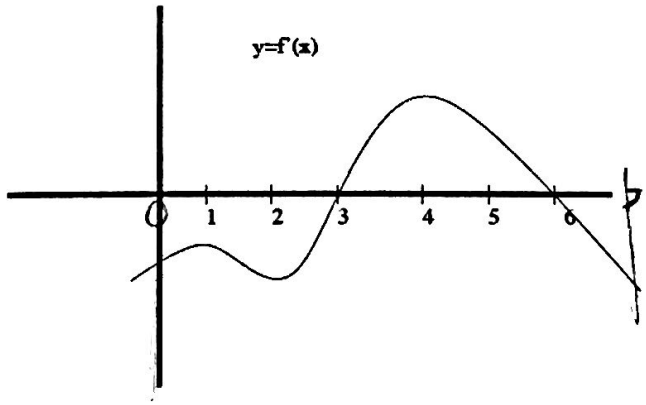

Inc $(-\infty, -2)(-2, 3)$

5. Given the graph of $f'(x)$, tell where Dec $(3, \infty)$
   a) $f(x)$ is increasing and decreasing.
   b) all relative maxima and minima are. max $x = 3$
   c) $f(x)$ is concave up and concave down.
   d) all points of inflection are. IP $x = -2$, $x = 1.5$

CU $(-2, 1.5)$ CD $(-\infty, -2)(1.5, \infty)$

6. The figure below is the graph of the *derivative* of a function $f$. Use this graph to answer the following questions about $f$ on the interval $0 \le x \le 7$. In each case, be sure to justify your answer.

   (a) On what subinterval(s) is $f(x)$ increasing? $(3, 6)$
   (b) On what subinterval(s) is $f''(x)$ positive? $(-\infty, 1)(2, 4)$
   (c) For what value of $x$ is $f''(x)$ the greatest? $x = 3$ ← steepest slope of $f'$
   (d) On what subinterval(s) is $f(x)$ concave up? $(-\infty, 1)(2, 4)$
   (e) For what value of $x$ is $f(x)$ increasing the fastest? $x = 4$ ← max of $f'$
   (f) For what value(s) of $x$ is $f''(x) = 0$? $x = 1, 2, 4$ ← slope of $f' = 0$
   (g) For what value(s) of $x$ might $f(x)$ be the smallest? $x = 3$ ← min of $f$



7. A function f is continuous over the interval described.

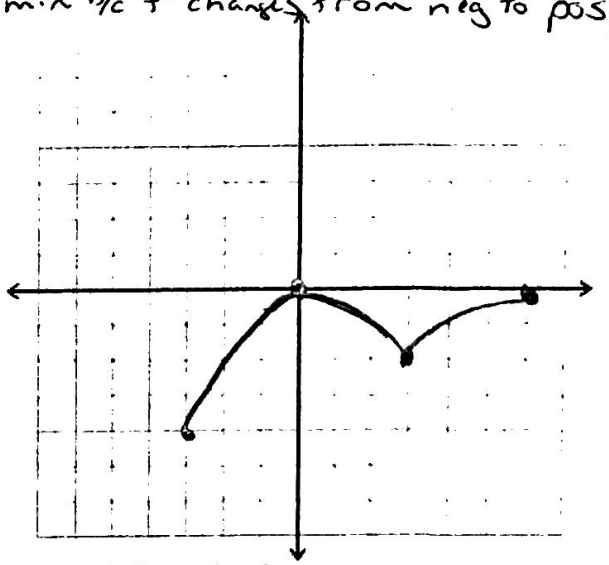
| x | -3 | -3 < x < 0 | 0 | 0 < x < 3 | 3 | 3 < x < 6 | 6 |
|---|---|---|---|---|---|---|---|
| $f$ | -4 | neg | 0 | neg | -2 | neg | 0 |
| $f'$ | 10 | pos | 0 | neg | DNE | pos | 2 |
| $f''$ | neg | neg | neg | neg | neg | neg | neg |

a) x = 0 is a relative max b/c f' changes from pos to neg, showing f(x) changing from increasing to decreasing. x = 3 is a relative min b/c f' changes from neg to pos, indicating f(x) changing from dec to inc.

a. Find the x-coordinates of all relative extrema on the domain [-3, 6]. Classify them as relative max or relative mins. Justify your answer.

b. Find the x-coordinates of any point of inflection on the domain [-3, 6]. Justify your answer.

c. Sketch a possible graph of f(x).



b) There is no pt of inflection. f'' remains negative which indicates f(x) remains concave down

8. A function f is even and continuous over the interval described.

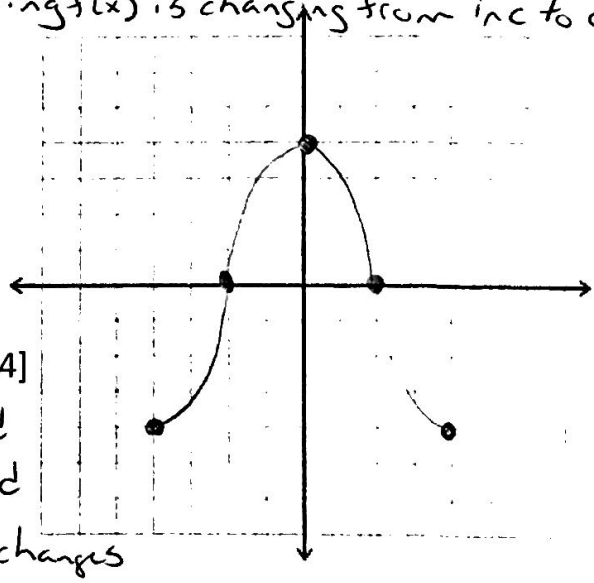
| x | 0 | 0 < x < 2 | 2 | 2 < x < 4 | 4 |
|---|---|---|---|---|---|
| $f$ | 4 | pos | 0 | neg | -4 |
| $f'$ | 0 | neg | -4 | neg | 0 |
| $f''$ | neg | neg | 0 | pos | pos |

a) f is even, so f is sym. to the y-axis. If f' is neg from (0,2), then f' is pos from (-2,0). So f' changes from pos to neg indicating f(x) is changing from inc to dec. Therefore x = 0 is a rel. max.

a. Find the x-coordinates of all relative extrema on the domain [-4, 4]. Classify them as relative max or relative mins. Justify your answer.

b. Find the x-coordinates of any point of inflection on the domain [-4, 4]. Justify your answer.

c. Sketch a possible graph of f(x) on the interval [-4, 4]



a) Since f is even, if f'' is neg from (0,2) and pos from (2,4), then f'' is pos from (-2,0) and neg from (-4,-2). If f'' changes sign, f(x) changes concavity. So x = -2 and x = 2 are inflection pts.