Applications of Integration

AP EXAM REVIEW

Motion Problems

$$\text{Velocity} = \frac{\text{change in position}}{\text{change in time}}$$

Average velocity = slope between two positions

Instantaneous velocity = velocity at instant in time
= derivative at a point

Speed = $|\text{velocity}|$

Sign of velocity shows direction.

Object is stationary when velocity = 0.

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{change in time}}$$
$$= \text{derivative of velocity}$$
$$= \text{2nd derivative of position}$$

Velocity is the derivative of position.

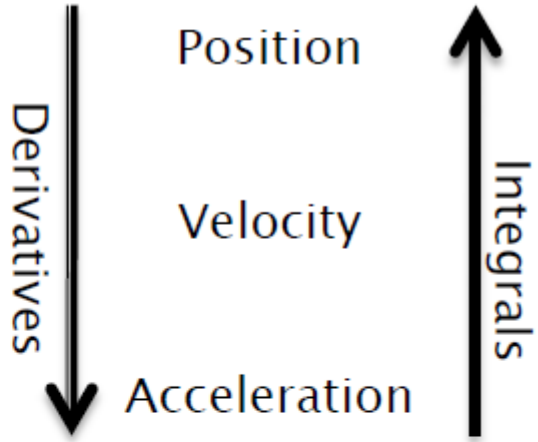
Position is the integral of velocity.

Displacement is the distance from an arbitrary starting point at the end of some interval. It is the difference between ending position and starting position.

$$\text{Displacement} = \int_a^b v(t)\,dt$$

Total distance is the how far an object travelled regardless of direction.

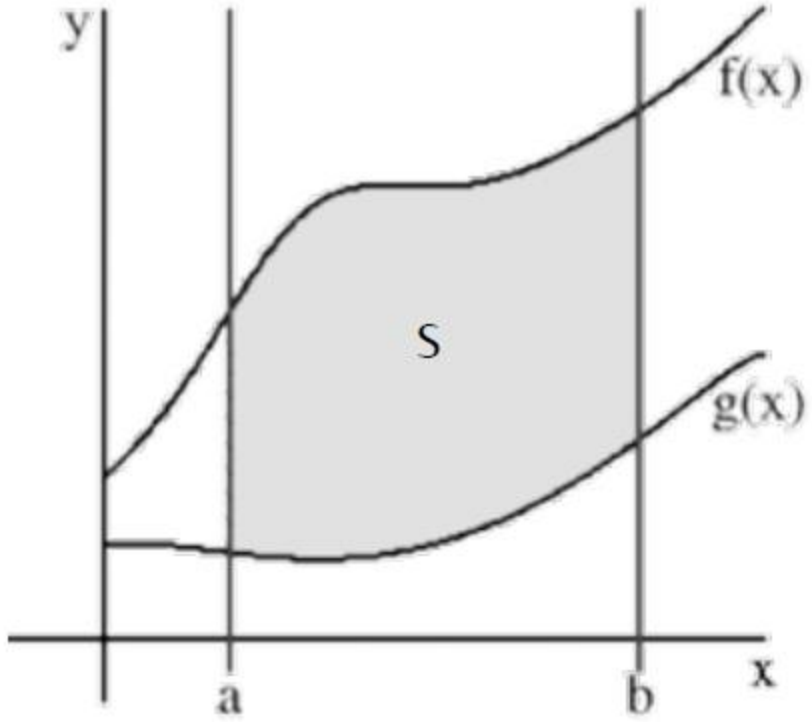
$$\text{Total Distance} = \int_a^b |v(t)|\,dt$$



Summary

To find	You
Displacement	$\int_a^b v(t)\,dt$
Total Distance	$\int_a^b \lvert v(t)\rvert\,dt$
Position at a specific time	Solve the indefinite integral and use given initial condition to find "c" $\int v(t)\,dt = P(t) + c$

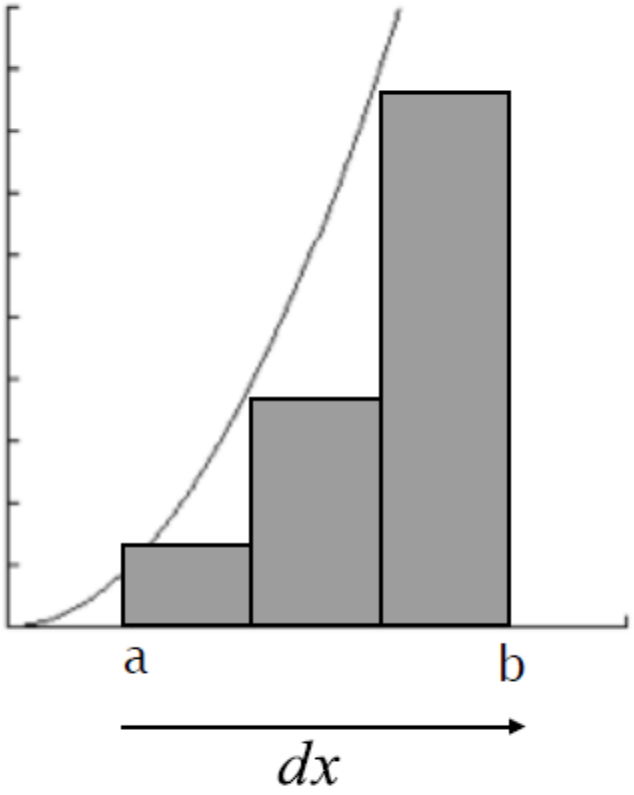
Area between curves



$$S = \int_a^b top - bottom$$

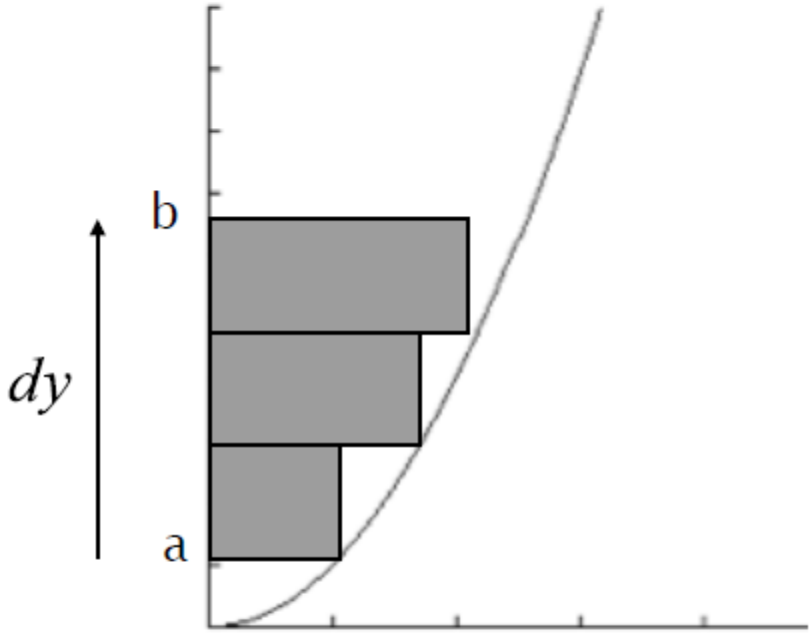
$$S = \int_a^b f(x) - g(x)\ dx$$

Why does *x* always have all the fun?



$$\int_{x\ value}^{x\ value} (f\ unction\ in\ x)\ dx$$

Let's integrate with respect to *y*



$$\int_{y\ value}^{y\ value} (f\ unction\ in\ y)\ dy$$

AP Tips

- Know how to find the points of intersection of two graphs. You can use your calculator's intersection or zero function. If a calculator is used, make sure to write the equation you are solving on your paper. "Bald" answers are not acceptable. Make sure to store the intersection points. However, if it is a quadratic or linear function or an easily factorable function, you should be able to solve for the intersection points algebraically.
- Remember area is always positive. If you solve the problem incorrectly and get a negative number, DO NOT just change the negative numbers to positive. Take the absolute value of all the steps that you already worked out. This will let you earn all the points for the problem.

Volume

Known cross-section
Disks
Washers

Area of Square = side ^{2}

Area of Semicircle = ½(diameter/2)^2 π

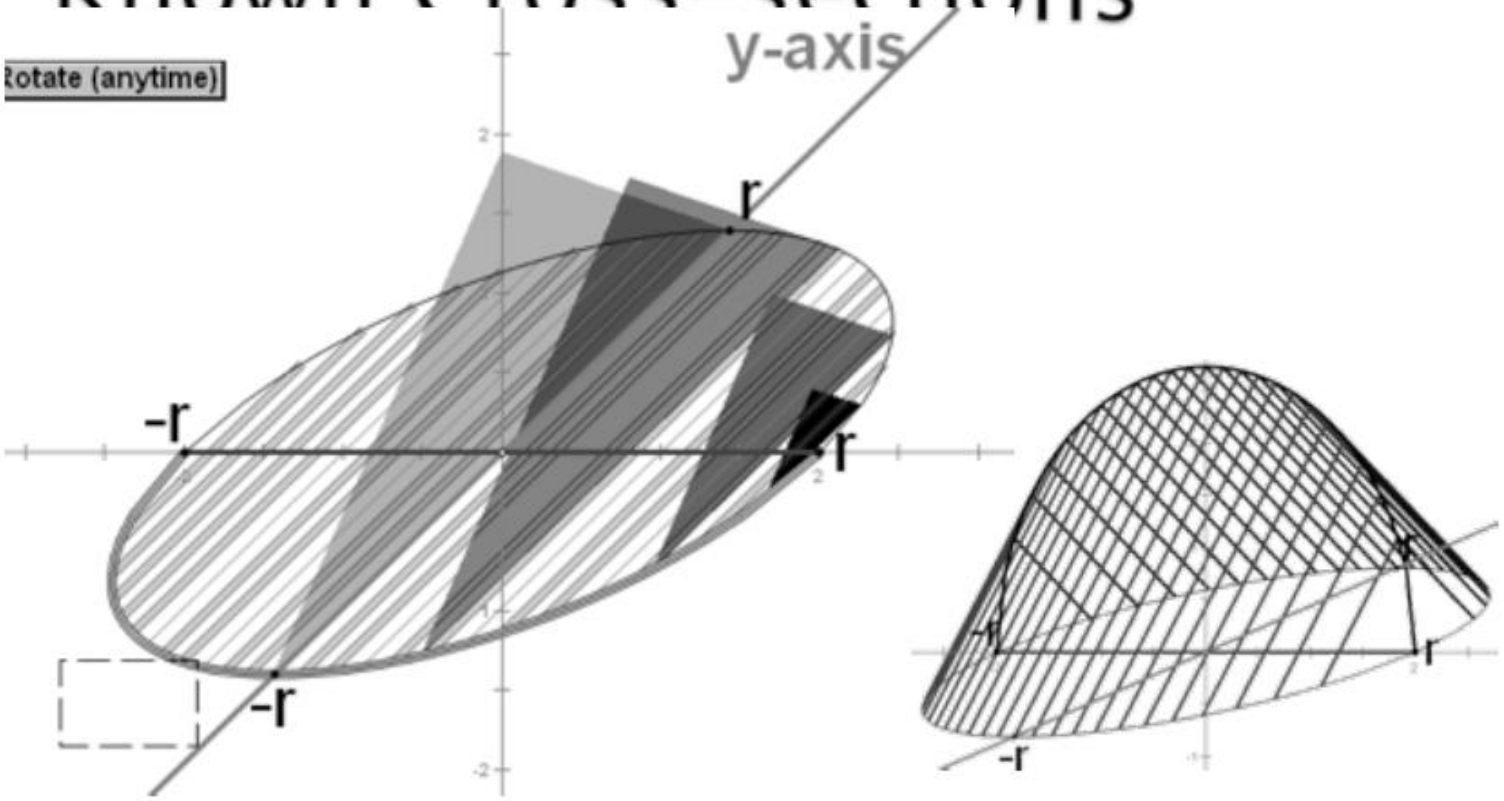
Equilateral Triangle

$$\text{Area} = \frac{\sqrt{3}edge^2}{4}$$

Right Triangle with hypotenuse touching base

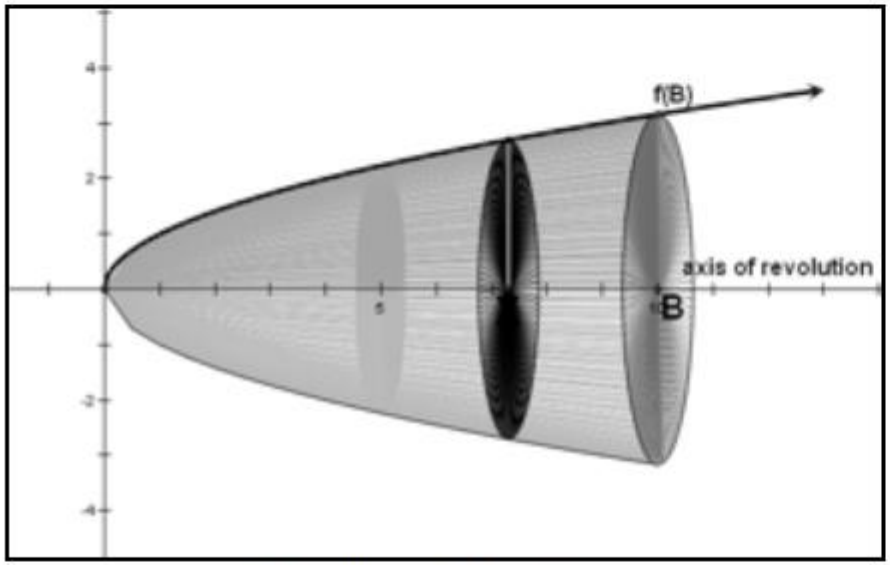
Area = hyp^2 / 4

Known Cross-sections

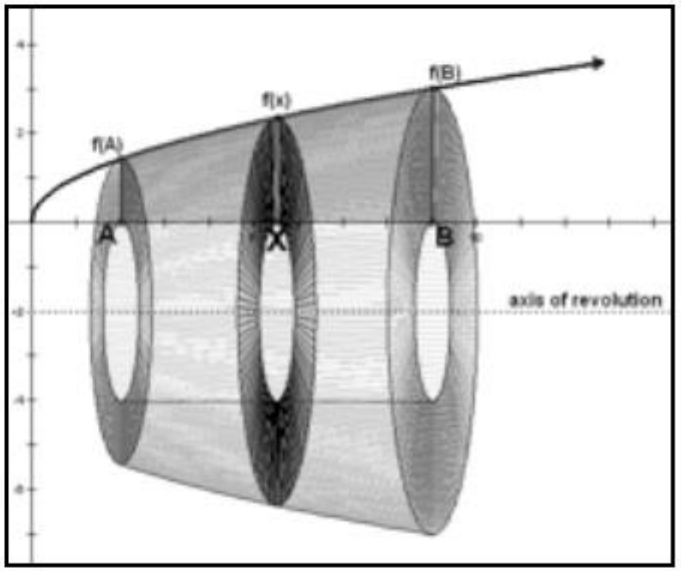


Be sure to check if cross-sections are perpendicular to the x-axis or y-axis!

Disks and Washers



Disk method

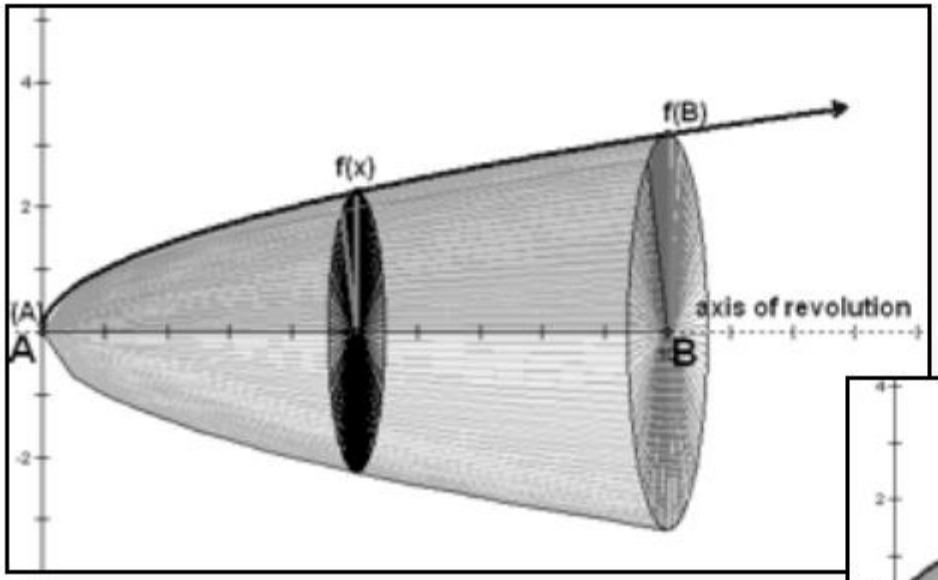


Washer method

$$= \pi \int_a^b r^2 \, dx$$

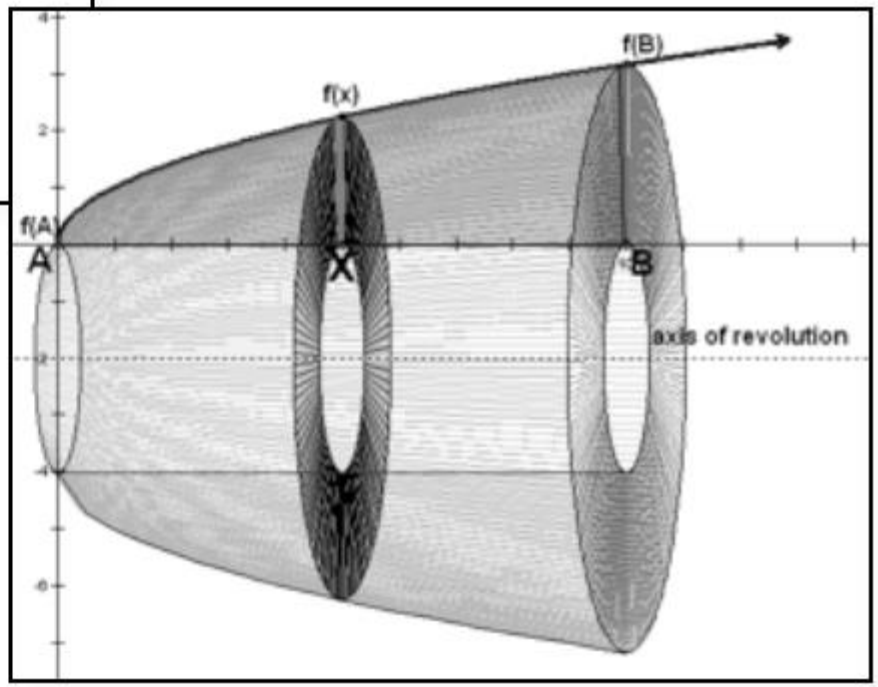
$$= \pi \int_a^b R^2 - r^2 \, dx$$

The axis of rotation is important



Same equations = same area being rotated.

But different axis of rotation = different solid



AP Tips

- Know how to calculate integrals on your calculator. Area and volume are two of the topics that generally require the ability to do this.
- The expectation is that you will set up the integral in the proper form, and use the calculator to evaluate the definite integral. Although you may be able to solve the integral by hand, don't. That will take valuable time and you could make an error along the way.
- The Shell method is on an AP topic, but it can be used on the exam.

Arc length of a function f(x) from x = a to x = b.

$$\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$