Area and Volume Problem Set

In questions 1-11, find the area of the region whose boundaries are given.

1. The curve of $y = x^2$, $y = 0$, $x = -1$, and $x = 2$.

(A) $\frac{11}{3}$ **(B)** $\frac{7}{3}$ **(C)** 3 **(D)** 5 **(E)** none of these

2. The parabola $y = x^2 - 3$ and the line $y = 1$.

(A) $\frac{8}{3}$ **(B)** 32 **(C)** $\frac{32}{3}$ **(D)** $\frac{16}{3}$ **(E)** none of these

3. The curve of $x = y^2 - 1$ and the y-axis.

(A) $\frac{4}{3}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{8}{3}$ **(D)** $\frac{1}{2}$ **(E)** none of these

4. The parabola $y^2 = x$ and the line $x + y = 2$.

(A) $\frac{5}{2}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{11}{6}$ **(D)** $\frac{9}{2}$ **(E)** $\frac{29}{6}$

5. The curve of $y = \frac{4}{x^2 + 4}$, the x-axis, and the vertical lines $x = -2$ and $x = 2$.

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{2}$ **(C)** 2π **(D)** π **(E)** none of these

6. The parabolas $x = y^2 - 5y$ and $x = 3y - y^2$.

(A) $\frac{32}{3}$ **(B)** $\frac{139}{6}$ **(C)** $\frac{64}{3}$ **(D)** $\frac{128}{3}$ **(E)** none of these

7. The curve of $y = \frac{2}{x}$ and $x + y = 3$.

(A) $\frac{1}{2} - 2\ln 2$ **(B)** $\frac{3}{2}$ **(C)** $\frac{1}{2} - \ln 4$

(D) $\frac{5}{2}$ **(E)** $\frac{3}{2} - \ln 4$

8. In the first quadrant, bounded below by the x-axis and above by the curves of $y = \sin x$ and $y = \cos x$.

(A) $2 - \sqrt{2}$ **(B)** $2 + \sqrt{2}$ **(C)** 2 **(D)** $\sqrt{2}$ **(E)** $2\sqrt{2}$

9. Bounded above by the curve $y = \sin x$ and below by $y = \cos x$ from $x = \frac{\pi}{4}$ to $x = \frac{5\pi}{4}$

(A) $2\sqrt{2}$ **(B)** $\frac{2}{\sqrt{2}}$ **(C)** $\frac{1}{2\sqrt{2}}$

(D) $2(\sqrt{2}-1)$ **(E)** $2(\sqrt{2}+1)$

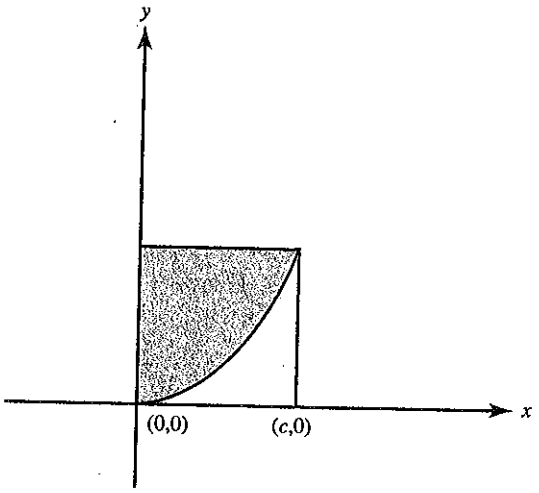
11. The curve of $y = x^3 - 2x^2 - 3x$ and the x-axis.

(A) $\frac{28}{3}$ **(B)** $\frac{79}{6}$ **(C)** $\frac{45}{4}$ **(D)** $\frac{71}{6}$ **(E)** none of these

12. The total area bounded by the cubic $x = y^3 - y$ and the line $x = 3y$ is equal to

(A) 4 **(B)** $\frac{16}{3}$ **(C)** 8 **(D)** $\frac{32}{3}$ **(E)** 16

13. The figure below shows part of the curve of $y = x^3$ and a rectangle with two vertices at $(0, 0)$ and $(c, 0)$. What is the ratio of the area of the rectangle to the shaded part of it above the cubic?



(A) 3:4 **(B)** 5:4 **(C)** 4:3 **(D)** 3:1 **(E)** 2:1

In questions 18-23, find the volume of the region whose boundaries are given and is rotated about the given line.

18. $y = x^2$, $x = 2$, and $y = 0$; about the x-axis.

(A) $\frac{64\pi}{3}$ **(B)** 8π **(C)** $\frac{8\pi}{3}$ **(D)** $\frac{128\pi}{5}$ **(E)** $\frac{32\pi}{5}$

19. $y = x^2$, $x = 2$, and $y = 0$; about the y-axis.

(A) $\frac{16\pi}{3}$ **(B)** 4π **(C)** $\frac{32\pi}{5}$ **(D)** 8π **(E)** $\frac{8\pi}{3}$

20. The first quadrant region bounded by $y = x^2$, the y-axis, and $y = 4$; about the y-axis.

(A) 8π **(B)** 4π **(C)** $\frac{64\pi}{3}$ **(D)** $\frac{32\pi}{3}$ **(E)** $\frac{16\pi}{3}$

21. $y = x^2$ and $y = 4$; about the x-axis.

(A) $\frac{64\pi}{5}$ **(B)** $\frac{512\pi}{15}$ **(C)** $\frac{256\pi}{5}$

(D) $\frac{128\pi}{5}$ **(E)** none of these

22. $y = x^2$ and $y = 4$; about the line $y = 4$.

(A) $\frac{256\pi}{15}$ **(B)** $\frac{256\pi}{5}$ **(C)** $\frac{512\pi}{5}$ **(D)** $\frac{512\pi}{15}$ **(E)** $\frac{64\pi}{3}$

23. An arch of $y = \sin x$ and the x-axis; about the x-axis.

(A) $\frac{\pi}{2}\left(\pi - \frac{1}{2}\right)$ **(B)** $\frac{\pi^2}{2}$ **(C)** $\frac{\pi^2}{4}$ **(D)** π^2 **(E)** $\pi(\pi - 1)$

26. The base of a solid is the region bounded by the parabola $x^2 = 8y$ and the line $y = 4$, and each plane section perpendicular to the y-axis is an equilateral triangle. The volume of the solid is

(A) $\frac{64\sqrt{3}}{3}$ **(B)** $64\sqrt{3}$ **(C)** $32\sqrt{3}$

(D) 32 **(E)** none of these

27. The base of a solid is the region bounded by $y = e^{-x}$, the x-axis, the y-axis, and the line $x = 1$. Each cross section perpendicular to the x-axis is a square. The volume of the solid is

(A) $\frac{e^2}{2}$ **(B)** $e^2 - 1$ **(C)** $1 - \frac{1}{e^2}$

(D) $\frac{e^2 - 1}{2}$ **(E)** $\frac{1}{2}\left(1 - \frac{1}{e^2}\right)$