

# WHAT DOES FUNCTION $f(x)=2x$ CALL HER MOTHER'S SISTER $F(x)=x^2$ ?

An equation involving derivatives is referred to as a **differential equation**.

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| <p>If <math>f'(x) = 4x^3</math>,<br/>a solution to this equation is <math>f(x) = x^4</math>,<br/>in fact there are an infinite number of solutions.<br/>Any function of the form <math>f(x) = x^4 + k</math>,<br/>where <math>k</math> is any constant, is a solution.<br/><math>\therefore f(x) = x^4 + k</math> is called the <b>general solution</b><br/>to the <b>differential equation</b> <math>f'(x) = 4x^3</math>.</p> | <p>If <math>f'(x) = 4x^3</math> and <math>f(2) = 10</math>, then<br/><math>f(x) = x^4 + k</math><br/><math>f(2) = 2^4 + k</math><br/><math>10 = 16 + k, k = -6</math><br/><math>\therefore f(x) = x^4 - 6</math><br/><math>f(x) = x^4 - 6</math> is called a <b>particular solution</b><br/>to the <b>differential equation</b> <math>f'(x) = 4x^3</math><br/>with <b>initial condition</b> <math>f(2) = 10</math>.</p> |
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Match each derivative  $f'(x)$  with a function  $f(x)$  below.

|                                  |                                   |                                   |                              |
|----------------------------------|-----------------------------------|-----------------------------------|------------------------------|
| 1) $f'(x) = 2x - 1$              | 2) $f'(x) = 3x^2 - 1$             | 3) $f'(x) = 3$                    | 4) $f'(x) = x^2 - x + 1$     |
| 5) $f'(x) = 5x^4 - 4x^3$         | 6) $f'(x) = x^4 - 4x + 5$         | 7) $f'(x) = 2x^4 - 2x^2$          | 8) $f'(x) = -x^{-2}$         |
| 9) $f'(x) = \frac{1}{2}x^{-1/2}$ | 10) $f'(x) = \frac{1}{3}x^{-2/3}$ | 11) $f'(x) = \frac{1}{2\sqrt{x}}$ | 12) $f'(x) = -\frac{1}{x^2}$ |

Functions (anti-derivatives).

|  |   |   |                             |
|--|---|---|-----------------------------|
| A. $f(x) = 3x + k$                         | B. $f(x) = x^3 + k$                             | D. $f(x) = x^2 - x + k$                             | E. $f(x) = \frac{1}{x} + k$ |
| F. $f(x) = \frac{1}{2}\sqrt{x} + k$        | H. $f(x) = \sqrt[3]{x} + k$                     | I. $f(x) = \sqrt{x} + k$                            | N. $f(x) = x^3 - x + k$     |
| R. $f(x) = x^5 - x^4 + k$                  | S. $f(x) = \frac{1}{3}x^2 - \frac{1}{2}x + k$   | T. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + x + k$ |                             |
| U. $f(x) = \frac{1}{5}x^5 - 2x^2 + 5x + k$ | V. $f(x) = \frac{2}{5}x^5 - \frac{2}{3}x^3 + k$ | W. $f(x) = \frac{2}{5}x^5 - 2x^3 + k$               |                             |

Match each differential equation that satisfies the given initial condition with a function below.

|                                   |   |   |
|-----------------------------------|---|---|
| 13) $f'(x) = 2x + 2, f(0) = 7$    | 14) $f'(x) = 2x + 2, f(1) = 7$                | 15) $f'(x) = 3x^2 - 3, f(0) = -1$           |
| 16) $f'(x) = x^4 - x^2, f(1) = 0$ | 17) $f'(x) = x^{-3} - \frac{1}{8}, f(2) = -1$ | 18) $f'(x) = \frac{3}{2}\sqrt{x}, f(4) = 4$ |

Particular solution to differential equations.

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| A. $f(x) = x^3 - 3x - 1$                                | E. $f(x) = x^2 + 2x + 4$                                   | G. $f(x) = x^2 + 2x$                        |
| H. $f(x) = x^3 - 3x$                                    | I. $f(x) = x^2 + 2x + 7$                                   | R. $f(x) = x\sqrt{x} - 4$                   |
| T. $f(x) = -\frac{1}{2x^2} - \frac{x}{8} - \frac{5}{8}$ | V. $f(x) = \frac{1}{5}x^5 - \frac{1}{3}x^3 + \frac{2}{15}$ | Z. $f(x) = \frac{1}{5}x^5 - \frac{1}{3}x^3$ |

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|    |    |   |
| 10 | 14 | 5 |

|   |   |   |   |    |    |
|---|---|---|---|----|----|
|   |   |   |   |    |    |
| 3 | 6 | 2 | 4 | 11 | 14 |

|   |    |    |    |    |    |    |   |   |   |
|---|----|----|----|----|----|----|---|---|---|
|   |    |    |    |    |    |    |   |   |   |
| 1 | 12 | 18 | 13 | 16 | 15 | 17 | 9 | 7 | 8 |