Chapter 2 Limits

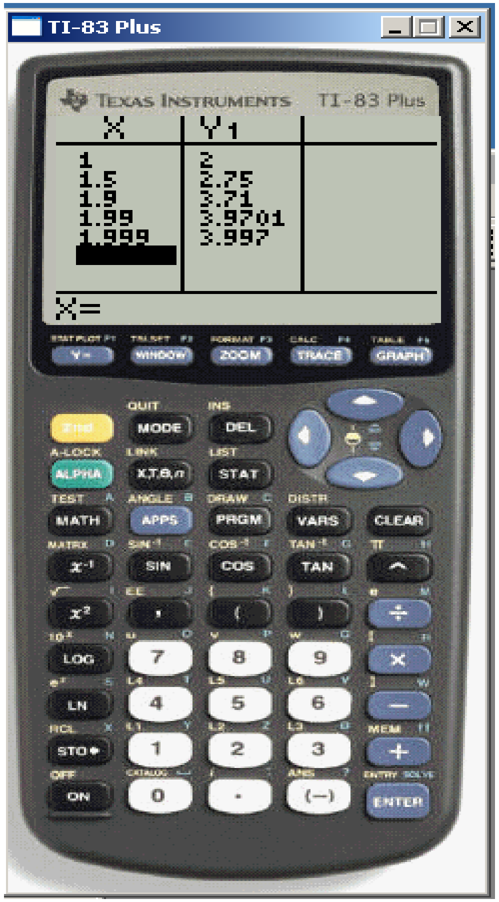
- Definition: The limit of f(x), as x approaches a, equals L if we can make the values of f(x) arbitrarily close to L by taking x to be sufficiently close to a, but not equal to a.

- We write limits in this form:

$$\lim_{x \to a} f(x) = L$$

- Use your calculator to look at the table for the function f(x) = $x^2 - x + 2$.

- We see when x is close to 2, f(x) is close to 4.



Therefore,

$$\lim_{x \to 2}(x^2 - x + 2) = 4$$

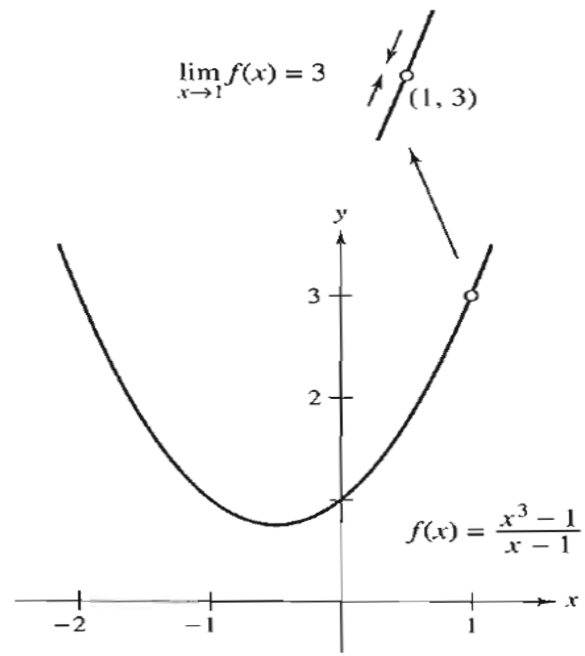
- Use your calculator to look at $f(x) = \frac{x^3 - 1}{x - 1}$

x approaches 1 from the left. → ← *x* approaches 1 from the right.

x	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25
$f(x)$	2.313	2.710	2.970	2.997	?	3.003	3.030	3.310	3.813

$f(x)$ approaches 3. → ← $f(x)$ approaches 3.

$$\lim_{x \to 1} \frac{x^3 - 1}{x - 1} = 3$$



- The existence of f(x) at x = c has no bearing on the existence of the limit of f(x) as x approaches c.

- Limits only care about what happens as you approach a given x-value.

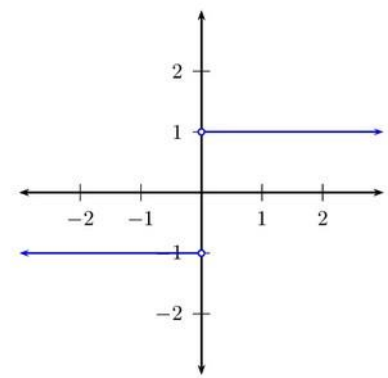
- Example: $f(x) = \begin{cases} 1, x \neq 2 \\ 0, x = 2 \end{cases}$

- $f(2) = 0$
- $\lim_{x \to 2} f(x) = 1$

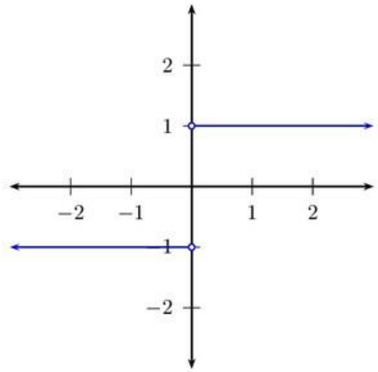
- What is $\lim_{x\to 0}\frac{|x|}{x}$?

What does the graph look like?

$$\frac{|x|}{x} = \begin{cases} \frac{x}{x} = 1, x > 0 \\ \frac{-x}{x} = -1, x < 0 \end{cases}$$



- It depends on which side you are coming from.



$$\lim_{x \to 0^-} \frac{|x|}{x} = -1$$

"left hand limit"

Approaches zero from the left, when x<0

$$\lim_{x \to 0^+} \frac{|x|}{x} = +1$$

"right hand limit"

Approaches zero from the right, when x>0

- $\lim_{x \to a} f(x) = L$ exists only if

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

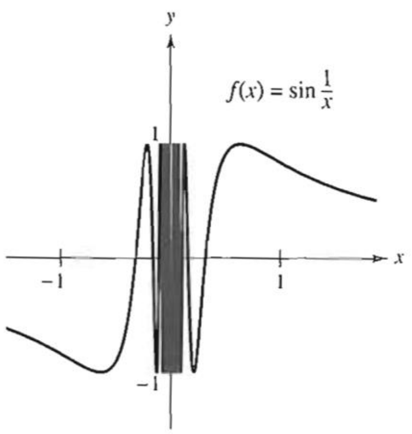
- Therefore, $\lim_{x \to 0} \frac{|x|}{x}$ does not exist because the left and right hand limits do not agree.

Example: $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$

- Let's look at it numerically and graphically with a x window set for [-2, 2]

x	2/π	2/(3π)	2/(5π)	2/(7π)	2/(9π)	2/(11π)
$\sin\left(\frac{1}{x}\right)$						

- F(x) oscillates between two fixed values, so $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.



Example: $\lim_{x \to 0} \frac{1}{x^2}$

- Graph the function. What is happening at 0?
- The function is approaching positive infinity. We say the function increases without bound.
- x = a is a vertical asymptote if a limit as x approaches a, from the left or right, equals positive or negative infinity.
- Limits that approach positive or negative infinity is the Calculus justification for why a vertical asymptote exists.

A limit needs to equal a specific value. Infinity is not a specific value, so technically, $\lim_{x \to 0} \frac{1}{x^2}$ does not exist at zero.

But we are concerned with the behavior of the graph, so when evaluating the limit of a function that increases or decreases without bound, we will describe the behavior as well as state the limit does not exist.

$$\lim_{x \to 0} \frac{1}{x^2} = \infty, DNE$$

Determine infinite limits numerically

$$\lim_{x \to 3} \frac{2x}{x-3}$$

- We learned in Pre-Cal that this function has a vertical asymptote at 3, but what is the function doing around 3?
- Let's look at the function from each side separately.

$$\lim_{x \to 3^+} \frac{2x}{x-3}$$

$$\lim_{x \to 3^-} \frac{2x}{x-3}$$

The left and right limits don't agree, so the limit does not exist.