46. (a) $\int_0^{\pi/2} \cos x\, dx = \sin x \Big]_0^{\pi/2} = 1$

(b) $\int_0^{\pi/2} \cos^2 x\, dx = \frac{1}{2}\int_0^{\pi/2} (1 + \cos 2x)\, dx = \frac{1}{2}\Big[x + \sin x \cos x\Big]_0^{\pi/2} = \frac{\pi}{4} \approx 0.78$

(c) Simpson's Rule $(n = 4)$

$$\int_0^{\pi/2} \cos(x^2)\, dx = \frac{\pi}{24}\left[1 + 4\cos\left(\frac{\pi}{8}\right)^2 + 2\cos\left(\frac{\pi}{4}\right)^2 + 4\cos\left(\frac{3\pi}{8}\right)^2 + \cos\left(\frac{\pi}{2}\right)^2\right] \approx 0.85$$

(d) Simpson's Rule $(n = 4)$

$$\int_0^{\pi/2} \cos\sqrt{x}\, dx = \frac{\pi}{24}\left[1 + 4\cos\sqrt{\pi/8} + 2\cos\sqrt{\pi/4} + 4\cos\sqrt{3\pi/8} + \cos\sqrt{\pi/2}\right] \approx 1.01$$

47. $\lim_{x\to 1}\left[\frac{(\ln x)^2}{x-1}\right] = \lim_{x\to 1}\left[\frac{2(1/x)\ln x}{1}\right] = 0$

48. $\lim_{x\to k}\left(\frac{x^{1/3} - k^{1/3}}{x-k}\right) = \lim_{x\to k}\left(\frac{1/3x^{-2/3}}{1}\right) = \frac{1}{3\sqrt[3]{k^2}}$

49. $\lim_{x\to\infty} \frac{e^{2x}}{x^2} = \lim_{x\to\infty}\frac{2e^{2x}}{2x} = \lim_{x\to\infty}\frac{4e^{2x}}{2} = \infty$

50. $\lim_{x\to 1^+}\left(\frac{2}{\ln x} - \frac{3}{x-1}\right) = \lim_{x\to 1^+}\left[\frac{2x - 2 - 3\ln x}{(\ln x)(x-1)}\right] = \lim_{x\to 1^+}\left[\frac{2 - (3/x)}{(x-1)(1/x) + \ln x}\right] = -\infty$

51. $y = \lim_{x\to\infty} (\ln x)^{2/x}$

$$\ln y = \lim_{x\to\infty}\frac{2\ln(\ln x)}{x} = \lim_{x\to\infty}\left[\frac{\frac{2}{x\ln x}}{1}\right] = 0$$

Since $\ln y = 0$, $y = 1$.

52. $y = \lim_{x\to 1}(x-1)^{\ln x}$

$$\ln y = \lim_{x\to 1}\left[(\ln x)\ln(x-1)\right] = \lim_{x\to 1}\left[\frac{\ln(x-1)}{\frac{1}{\ln x}}\right] = \lim_{x\to 1}\left[\frac{\frac{1}{x-1}}{\left(\frac{1}{x}\right)\frac{-1}{\ln^2 x}}\right] = \lim_{x\to 1}\left[\frac{-\ln^2 x}{\frac{x-1}{x}}\right]$$

$$= \lim_{x\to 1}\left[\frac{-2(1/x)(\ln x)}{1/x^2}\right]$$

$$= \lim_{x\to 1} 2x(\ln x) = 0$$

Since $\ln y = 0$, $y = 1$.

53. $\lim_{n\to\infty} 1000\left(1 + \frac{0.09}{n}\right)^n = 1000e^{0.09} \approx 1094.17$

54. $\lim_{x\to\infty} xe^{-x^2} = \lim_{x\to\infty}\frac{x}{e^{x^2}} = \lim_{x\to\infty}\frac{1}{2xe^{x^2}} = 0$

55. $\lim_{x\to 0} \csc 3x \tan \pi x = \lim_{x\to 0}\frac{\tan \pi x}{\sin 3x}$

$$= \lim_{x\to 0}\frac{\pi \sec^2 \pi x}{3\cos 3x} = \frac{\pi}{3}$$

56. $\lim_{x\to 0}\frac{\sin \pi x}{\sin 2\pi x} = \lim_{x\to 0}\frac{\pi\cos\pi x}{2\pi\cos 2\pi x} = \frac{\pi}{2\pi} = \frac{1}{2}$

57. $s = \int_0^{\pi}\sqrt{1 + \cos^2 x}\, dx \approx 3.82$

58. $s = \int_0^{\pi}\sqrt{1 + \sin^2 2x}\, dx \approx 3.82$