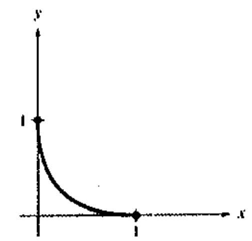
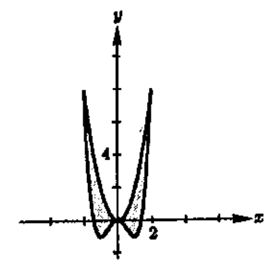
13. $y = (1-\sqrt{x})^2$

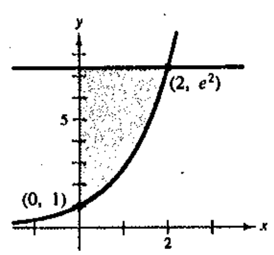
$$A = \int_0^1 (1-\sqrt{x})^2\,dx$$
$$= \int_0^1 (1 - 2x^{1/2} + x)\,dx$$
$$= \left[x - \frac{4}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^1 = \frac{1}{6}$$



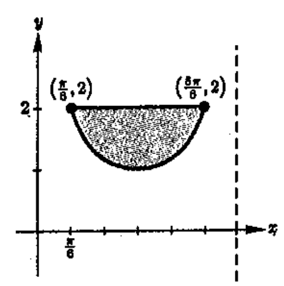
14. $$A = 2\int_0^2 [2x^2 - (x^4 - 2x^2)]\,dx$$
$$= 2\int_0^2 (4x^2 - x^4)\,dx$$
$$= 2\left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2 = \frac{128}{15}$$



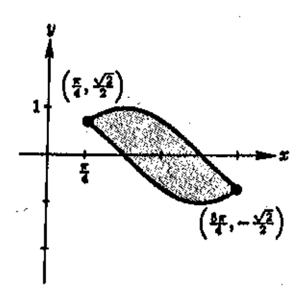
15. $$A = \int_0^2 (e^2 - e^x)\,dx$$
$$= \left[xe^2 - e^x\right]_0^2$$
$$= e^2 + 1$$



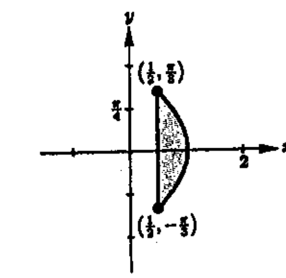
16. $$A = 2\int_{\pi/6}^{\pi/2} (2 - \csc x)\,dx$$
$$= 2\left[2x - \ln|\csc x - \cot x|\right]_{\pi/6}^{\pi/2}$$
$$= 2\left([\pi - 0] - \left[\frac{\pi}{3} - \ln(2-\sqrt{3})\right]\right)$$
$$= 2\left[\frac{2\pi}{3} + \ln(2-\sqrt{3})\right] \approx 1.555$$



17. $$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\,dx$$
$$= (-\cos x - \sin x)\Big]_{\pi/4}^{5\pi/4}$$
$$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

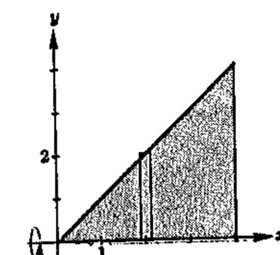


18. $$A = \int_{\pi/3}^{5\pi/3} \left(\frac{1}{2} - \cos y\right) dy + \int_{5\pi/3}^{7\pi/3} \left(\cos y - \frac{1}{2}\right) dy$$
$$= \left[\frac{y}{2} - \sin y\right]_{\pi/3}^{5\pi/3} + \left[\sin y - \frac{y}{2}\right]_{5\pi/3}^{7\pi/3}$$
$$= \frac{\pi}{3} + 2\sqrt{3}$$



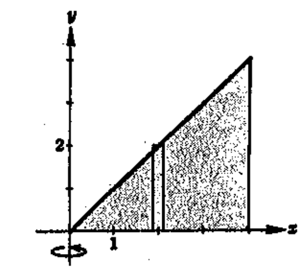
19. (a) **Disc**

$$V = \pi\int_0^4 x^2\,dx = \left[\frac{\pi x^3}{3}\right]_0^4 = \frac{64\pi}{3}$$



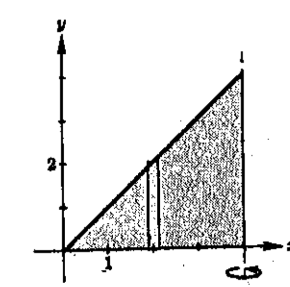
(b) **Shell**

$$V = 2\pi\int_0^4 x^2\,dx = \left[\frac{2\pi}{3}x^3\right]_0^4 = \frac{128\pi}{3}$$



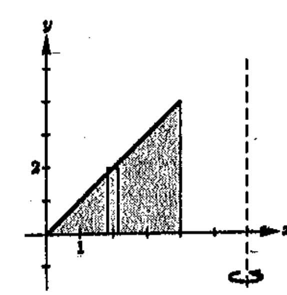
(c) **Shell**

$$V = 2\pi\int_0^4 (4-x)x\,dx$$
$$= 2\pi\int_0^4 (4x - x^2)\,dx$$
$$= 2\pi\left[2x^2 - \frac{x^3}{3}\right]_0^4 = \frac{64\pi}{3}$$



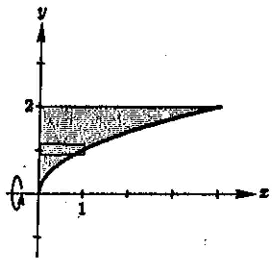
(d) **Shell**

$$V = 2\pi\int_0^4 (6-x)x\,dx$$
$$= 2\pi\int_0^4 (6x - x^2)\,dx$$
$$= 2\pi\left[3x^2 - \frac{1}{3}x^3\right]_0^4 = \frac{160\pi}{3}$$



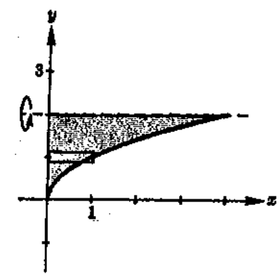
20. (a) **Shell**

$$V = 2\pi\int_0^2 y^3\,dy = \frac{\pi}{2}y^4\Big]_0^2 = 8\pi$$



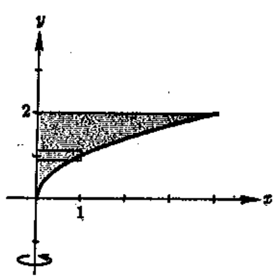
(b) **Shell**

$$V = 2\pi\int_0^2 (2-y)y^2\,dy$$
$$= 2\pi\int_0^2 (2y^2 - y^3)\,dy$$
$$= 2\pi\left[\frac{2}{3}y^3 - \frac{1}{4}y^4\right]_0^2 = \frac{8\pi}{3}$$



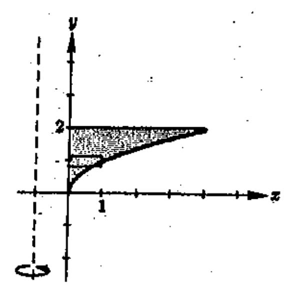
(c) **Disc**

$$V = \pi\int_0^2 y^4\,dy$$
$$= \frac{\pi}{5}y^5\Big]_0^2 = \frac{32\pi}{5}$$



(d) **Disc**

$$V = \pi\int_0^2 [(y^2+1)^2 - 1^2]\,dy$$
$$= \pi\int_0^2 (y^4 + 2y^2)\,dy$$
$$= \pi\left[\frac{1}{5}y^5 + \frac{2}{3}y^3\right]_0^2 = \frac{176\pi}{15}$$



21. From Exercise 22, letting $a = 4$ and $b = 3$ we have

(a) $V = \frac{4}{3}\pi(4^2)(3) = 64\pi$

(b) $V = \frac{4}{3}\pi(4)(3)^2 = 48\pi$

37. $$A = \int_0^a (\sqrt{a} - \sqrt{x})^2\,dx = \int_0^a (a - 2\sqrt{a}\,x^{1/2} + x)\,dx = \left[ax - \frac{4}{3}\sqrt{a}\,x^{3/2} + \frac{1}{2}x^2\right]_0^a = \frac{a^2}{6}$$

$$\frac{1}{A} = \frac{6}{a^2}$$

$$\bar{x} = \frac{6}{a^2}\int_0^a x(\sqrt{a} - \sqrt{x})^2\,dx = \frac{6}{a^2}\int_0^a (ax - 2\sqrt{a}\,x^{3/2} + x^2)\,dx = \frac{6}{a^2}\left[\frac{ax^2}{2} - \frac{4}{5}\sqrt{a}\,x^{5/2} + \frac{1}{3}x^3\right]_0^a = \frac{a}{5}$$

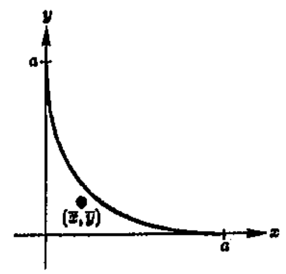
$$\bar{y} = \left(\frac{6}{a^2}\right)\frac{1}{2}\int_0^a (\sqrt{a} - \sqrt{x})^4\,dx$$

$$= \frac{3}{a^2}\int_0^a (a^2 - 4a^{3/2}x^{1/2} + 6ax - 4a^{1/2}x^{3/2} + x^2)\,dx$$

$$= \frac{3}{a^2}\left[a^2x - \frac{8}{3}a^{3/2}x^{3/2} + 3ax^2 - \frac{8}{5}a^{1/2}x^{5/2} + \frac{1}{3}x^3\right]_0^a$$

$$= \frac{a}{5}$$

$$(\bar{x}, \bar{y}) = \left(\frac{a}{5}, \frac{a}{5}\right)$$



38. $$A = \int_{-1}^3 [(2x+3) - x^2]\,dx = \left[x^2 + 3x - \frac{1}{3}x^3\right]_{-1}^3 = \frac{32}{3}$$

$$\frac{1}{A} = \frac{3}{32}$$

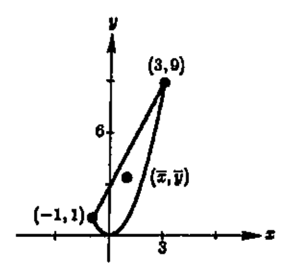
$$\bar{x} = \frac{3}{32}\int_{-1}^3 x(2x + 3 - x^2)\,dx = \frac{3}{32}\int_{-1}^3 (3x + 2x^2 - x^3)\,dx = \frac{3}{32}\left[\frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_{-1}^3 = 1$$

$$\bar{y} = \left(\frac{3}{32}\right)\frac{1}{2}\int_{-1}^3 [(2x+3)^2 - x^4]\,dx$$

$$= \frac{3}{64}\int_{-1}^3 (9 + 12x + 4x^2 - x^4)\,dx$$

$$= \frac{3}{64}\left[9x + 6x^2 + \frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{-1}^3 = \frac{17}{5}$$

$$(\bar{x}, \bar{y}) = \left(1, \frac{17}{5}\right)$$



39. By symmetry, $\bar{x} = 0$.

$$A = 2\int_0^1 (a^2 - x^2)\,dx = 2\left[a^2x - \frac{x^3}{3}\right]_0^a = \frac{4a^3}{3}$$

$$\frac{1}{A} = \frac{3}{4a^3}$$

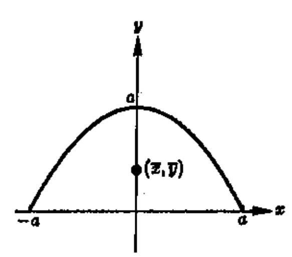
$$\bar{y} = \left(\frac{3}{4a^3}\right)\frac{1}{2}\int_{-a}^a (a^2 - x^2)^2\,dx$$

$$= \frac{6}{8a^3}\int_0^a (a^4 - 2a^2x^2 + x^4)\,dx$$

$$= \frac{6}{8a^3}\left[a^4x - \frac{2a^2}{3}x^3 + \frac{1}{5}x^5\right]_0^a$$

$$= \frac{6}{8a^3}\left(a^5 - \frac{2}{3}a^5 + \frac{1}{5}a^5\right) = \frac{2a^2}{5}$$

$$(\bar{x}, \bar{y}) = \left(0, \frac{2a^2}{5}\right)$$



40. $$A = \int_0^8 \left(x^{2/3} - \frac{1}{2}x\right)dx = \left[\frac{3}{5}x^{5/3} - \frac{1}{4}x^2\right]_0^8 = \frac{16}{5}$$

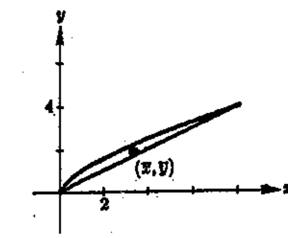
$$\frac{1}{A} = \frac{5}{16}$$

$$\bar{x} = \frac{5}{16}\int_0^8 x\left(x^{2/3} - \frac{1}{2}x\right)dx$$

$$= \frac{5}{16}\left[\frac{3}{8}x^{8/3} - \frac{1}{6}x^3\right]_0^8 = \frac{10}{3}$$

$$\bar{y} = \left(\frac{5}{16}\right)\frac{1}{2}\int_0^8 \left(x^{4/3} - \frac{1}{4}x^2\right)dx = \frac{1}{2}\left(\frac{5}{16}\right)\left[\frac{3}{7}x^{7/3} - \frac{1}{12}x^3\right]_0^8 = \frac{40}{21}$$

$$(\bar{x}, \bar{y}) = \left(\frac{10}{3}, \frac{40}{21}\right)$$



41. $$y = \sqrt{4 - x^2}$$

$$y' = \frac{-x}{\sqrt{4 - x^2}}$$

$$1 + (y')^2 = \frac{4}{4 - x^2}$$

$$s = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{\frac{4}{4 - x^2}}\,dx = 2\int_0^{\sqrt{3}} \frac{2}{\sqrt{4 - x^2}}\,dx$$

$$= 4\arcsin\frac{x}{2}\Big]_0^{\sqrt{3}}$$

$$= 4\arcsin\frac{\sqrt{3}}{2} = \frac{4\pi}{3}$$

$$\frac{2\pi r}{3} = \frac{2\pi(2)}{3} = \frac{4\pi}{3} = s$$

42. $$y = \frac{x^3}{6} + \frac{1}{2x}$$

$$y' = \frac{1}{2}x^2 - \frac{1}{2x^2}$$

$$1 + (y')^2 = \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)^2$$

$$s = \int_1^3 \left(\frac{1}{2}x^2 + \frac{1}{2x^2}\right)dx$$

$$= \left[\frac{1}{6}x^3 - \frac{1}{2x}\right]_1^3 = \frac{14}{3}$$

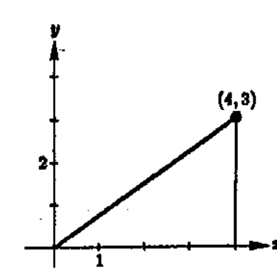
43. $$y = \frac{3}{4}x$$

$$y' = \frac{3}{4}$$

$$1 + (y')^2 = \frac{25}{16}$$

$$S = 2\pi\int_0^4 \left(\frac{3}{4}x\right)\sqrt{\frac{25}{16}}\,dx = \left(\frac{15\pi}{8}\right)\frac{x^2}{2}\Big]_0^4$$

$$= 15\pi$$



44. From Exercise 21(a) we have

$$V = 64\pi\ \text{ft}^3$$

$$\frac{1}{4}V = 16\pi.$$

Disc:

$$\pi\int_{-3}^{y_0} \frac{16}{9}(9 - y^2)\,dy = 16\pi$$

$$\frac{1}{9}\int_{-3}^{y_0} (9 - y^2)\,dy = 1$$

$$\left[9y - \frac{1}{3}y^3\right]_{-3}^{y_0} = 9$$

$$\left(9y_0 - \frac{1}{3}{y_0}^3\right) - (-27 + 9) = 9$$

$${y_0}^3 - 27y_0 - 27 = 0$$

By Newton's Method, $y_0 \approx -1.042$ and the depth of the gasoline is $3 - 1.042 = 1.958$ ft.

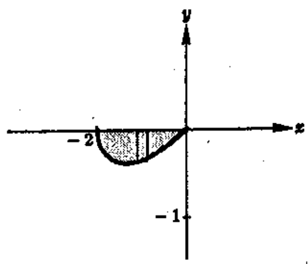
45. Since $y \le 0$, $A = -\int_{-1}^{0} x\sqrt{x+1}\,dx$.

By the substitution method we obtain

$$A = -\int_{-1}^{0}\left[(x+1)^{3/2} - (x+1)^{1/2}\right]dx$$

$$= \left[-\frac{2}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2}\right]_{-1}^{0}$$

$$= \frac{4}{15}.$$



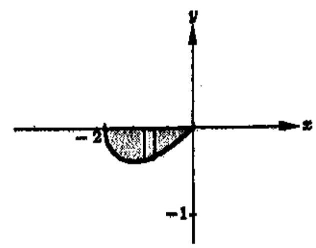
46. Disc

$$V = \pi\int_{-1}^{0} x^2(x+1)\,dx$$

$$= \pi\int_{-1}^{0}(x^3 + x^2)\,dx$$

$$= \pi\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_{-1}^{0}$$

$$= \frac{\pi}{12}$$



47. Shell

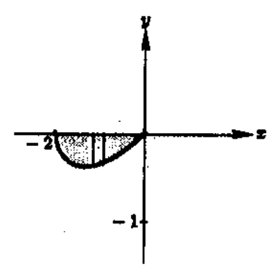
$u = \sqrt{x+1}$

$x = u^2 - 1$

$dx = 2u\,du$

$$V = 2\pi\int_{-1}^{0} x^2\sqrt{x+1}\,dx = 4\pi\int_{0}^{1}(u^2-1)^2u^2\,du = 4\pi\int_{0}^{1}(u^6 - 2u^4 + u^2)\,du$$

$$= 4\pi\left[\frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3\right]_{0}^{1} = \frac{32\pi}{105}$$



48.

$$y = 2\sqrt{x}$$

$$y' = \frac{1}{\sqrt{x}}$$

$$1 + (y')^2 = 1 + \frac{1}{x} = \frac{x+1}{x}$$

$$S = 2\pi\int_{0}^{3} 2\sqrt{x}\sqrt{\frac{x+1}{x}}\,dx = 4\pi\int_{0}^{3}\sqrt{x+1}\,dx = 4\pi\left(\frac{2}{3}\right)(x+1)^{3/2}\Big]_{0}^{3} = \frac{56\pi}{3}$$

49.
$$f(x) = \frac{4}{5}x^{5/4}$$
$$f'(x) = x^{1/4}$$
$$1 + [f'(x)]^2 = 1 + \sqrt{x}$$
$$u = 1 + \sqrt{x}$$
$$x = (u-1)^2$$
$$dx = 2(u-1)\,du$$
$$s = \int_0^4 \sqrt{1+\sqrt{x}}\,dx = 2\int_1^3 \sqrt{u}(u-1)\,du = 2\int_1^3 (u^{3/2} - u^{1/2})\,du = 2\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^3$$
$$= \frac{4}{15}u^{3/2}(3u-5)\Big]_1^3$$
$$= \frac{8}{15}(1 + 6\sqrt{3}) \approx 6.076$$

50. **Shell**
$$u = \sqrt{x-2}$$
$$x = u^2 + 2$$
$$dx = 2u\,du$$
$$V = 2\pi\int_2^6 \frac{x}{1+\sqrt{x-2}}\,dx = 4\pi\int_0^2 \frac{(u^2+2)u}{1+u}\,du = 4\pi\int_0^2 \frac{u^3+2u}{1+u}\,du$$
$$= 4\pi\int_0^2 \left(u^2 - u + 3 - \frac{3}{1+u}\right) du$$
$$= 4\pi\left[\frac{1}{3}u^3 - \frac{1}{2}u^2 + 3u - 3\ln(1+u)\right]_0^2$$
$$= \frac{4\pi}{3}(20 - 9\ln 3)$$