Worksheet 4. What You Need to Know About Motion Along the *x*-axis (Part 2)

1. Speed is the absolute value of _____.

2. If the velocity and acceleration have the same sign (both positive or both negative), then speed is _____.

3. If the velocity and acceleration are _____ in sign (one is positive and the other is negative), then speed is decreasing.

There are three ways to use an integral in the study of motion that are easily confused. Watch out!

4. $\int v(t)\,dt$ is an _____ integral. It will give you an expression for _____ at time *t*. Don't forget that you will have a _____, the value of which can be determined if you know a position value at a particular time.

5. $\int_{t_1}^{t_2} v(t)\,dt$ is a _____ integral and so the answer will be a _____. The number represents the change in _____ over the time interval. By the Fundamental Theorem of Calculus, since $v(t) = x'(t)$, the integral will yield $x(t_2) - x(t_1)$. This is also known as displacement. The answer can be positive or _____ depending upon if the particle lands to the _____ or left of its original starting position.

6. $\int_{t_1}^{t_2} |v(t)|\,dt$ is also a _____ integral and so the answer will be a number. The number represents the _____ traveled by the particle over the time interval. The answer should always be _____.

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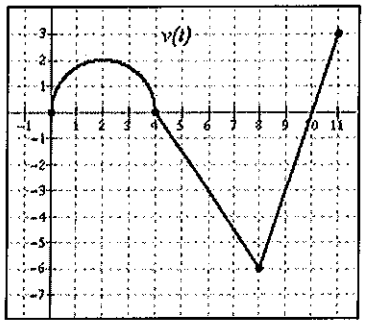
Worksheet 5. Sample Practice Problems for the Topic of Motion (Part 2)

Example 1 (graphical).

The graph to the right shows the velocity, $v(t)$, of a particle moving along the x-axis for $0 \le t \le 11$. It consists of a semicircle and two line segments. Use the graph and your knowledge of motion to answer the following questions.



1. At what time t, $0 \le t \le 11$, is the speed of the particle the greatest?

2. At which of the times, $t = 2$, $t = 6$ or $t = 9$, is the acceleration of the particle the greatest? Explain your answer.

3. Over what time intervals is the particle moving to the left? Explain your answer.

4. Over what time intervals is the speed of the particle decreasing? Explain your answer.

5. Find the total distance traveled by the particle over the time interval $0 \le t \le 11$.

6. Find the value of $\int_0^{11} v(t)\,dt$ and explain the meaning of this integral in the context of the problem.

7. If the initial position of the particle is $x(0) = 2$, find the position of the particle at time $t = 11$.

Example 2 (analytic/graphical/calculator active).

The rate of change, in kilometers per hour, of the altitude of a hot air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for time $0 \le t \le 4$, where t is measured in hours. Assume the balloon is initially at ground level.

1. For what values of t, $0 \le t \le 4$, is the altitude of the balloon decreasing?

2. Find the value of $r'(2)$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

3. What is the altitude of the balloon when it is closest to the ground during the time interval $2 \le t \le 4$?

4. Find the value of $\int_0^4 r(t)\,dt$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

5. Find the value of $\int_0^4 |r(t)|\,dt$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

6. What is the maximum altitude of the balloon during the time interval $0 \le t \le 4$?

Example 3 (numerical).

The table below gives values for the velocity and acceleration of a particle moving along the x-axis for selected values of time t. Both velocity and acceleration are differentiable functions of time t. The velocity is decreasing for all values of t, $0 \le t \le 10$. Use the data in the table to answer the questions that follow.

Time, t	0	2	6	10
Velocity, $v(t)$	5	3	-1	-8
Acceleration, $a(t)$	0	-1	-3	-5

1. Is there a time t when the particle is at rest? Explain your answer.

2. At what time indicated in the table is the speed of the particle decreasing? Explain your answer.

3. Use a left Riemann sum to approximate $\int_0^{10} v(t)\,dt$. Show the computations you use to arrive at your answer. Explain the meaning of the definite integral in the context of the problem.

4. Is the approximation found in part (3) greater than or less than the actual value of the definite integral shown below? Explain your reasoning.

$$\int_0^{10} v(t)\,dt$$

5. Approximate the value of $\int_0^{10} |v(t)|\,dt$ using a trapezoidal approximation with the three sub-intervals indicated by the values in the table. Show the computations you use to arrive at your answer. Explain the meaning of the definite integral in the context of the problem.

6. Determine the value of $\int_0^{10} a(t)\,dt$. Explain the meaning of the definite integral in the context of the problem.

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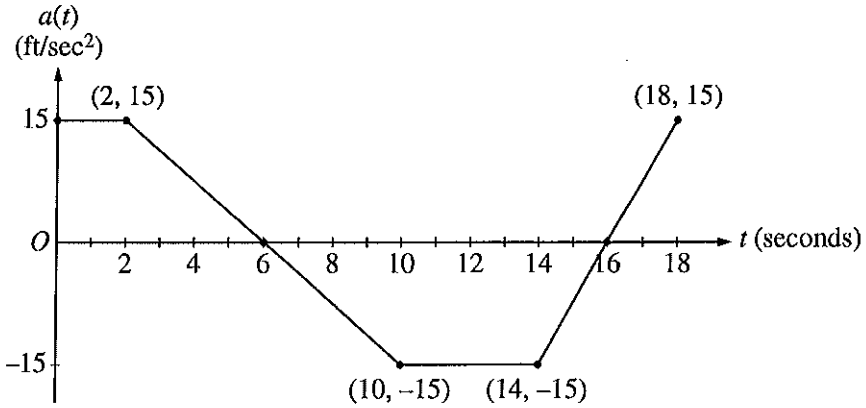

3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

   (a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

   (b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

   (c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

   (d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

END OF PART A OF SECTION II