Section 2.5 Continuity

Definition:

A function f(x) is continuous at c if

1. f(c) is defined
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

3 common types of discontinuity:

1. Removable discontinuity (holes)
2. Infinite discontinuity (vertical asymptotes)
3. Jump discontinuity (piecewise functions)

On worksheet:

- jump discontinuity at $x = 0$
- removable discontinuity at $x = 1$ and $x = 2$
- Infinite discontinuity at $x = 4$

Example 1: Is f(x) continuous at x = 3? Justify your answer.

- $f(x) = \begin{cases} 2x, & x < 3 \\ x + 3, & x \geq 3 \end{cases}$

Example 2: Is f(x) continuous at x = 4? Justify your answer.

- $f(x) = \begin{cases} 3x, & x < 4 \\ x + 5, & x \geq 4 \end{cases}$

What does k equal so that f(x) is continuous?

- $f(x) = \begin{cases} x^2 + 2, & x > 2 \\ -2x^2 - k, & x \le 2 \end{cases}$

At an endpoint:

- If $\lim_{x \to a+} f(x) = f(a)$ or $\lim_{x \to b-} f(x) = f(b)$,

a function can be continuous from the right at a or continuous from the left at b.

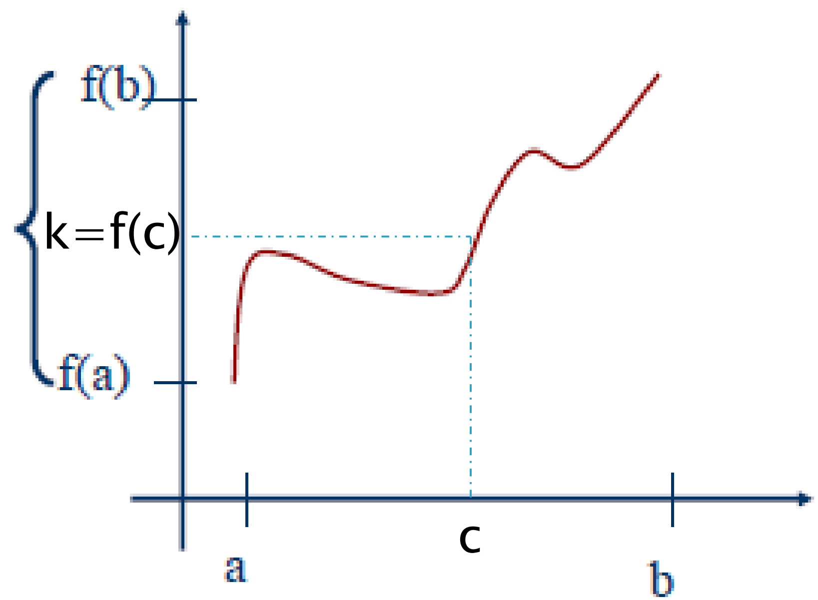
A function can be continuous on an interval.

- $f(x) = 1 - \sqrt{1 - x^2}$

Intermediate Value Theorem

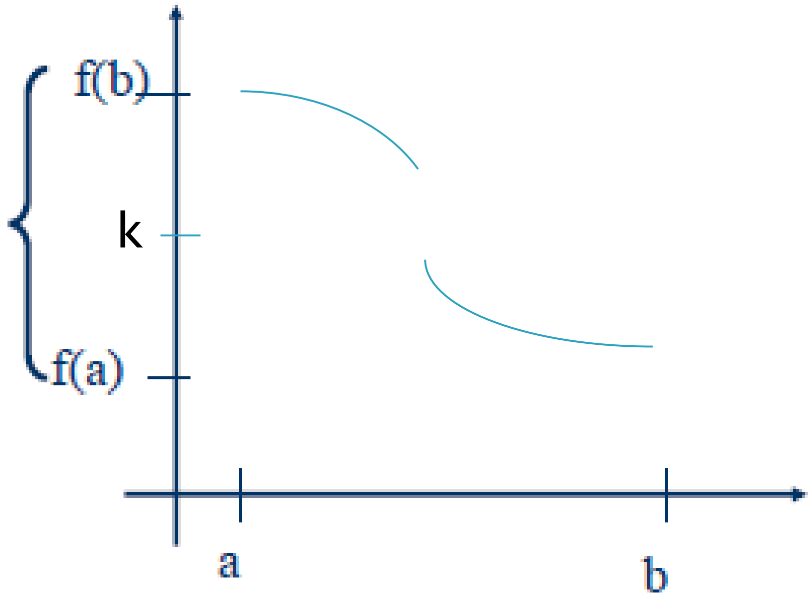
If f(x) is continuous on [a,b], and k is between f(a) and f(b), then k = f(c) for some c in [a,b].

In other words, if f(x) is continuous on [a,b], it takes on all y values between f(a) and f(b).



Function must be continuous

No c between a and b exists such that f(c)=k.



- The theorem can be used to locate or verify solutions or “zeros” of continuous functions.

- A solution occurs when the function crosses the x-axis or $f(x)=0$. Therefore if $f(a)$ and $f(b)$ have different signs, the theorem guarantees the existence of at least on zero on the interval of $[a,b]$.

Verify that $x^3 + 2x - 1 = 0$ has at least one solution in the interval [0,1].

$$f(x) = x^3 + 2x - 1$$

f(0) = –1

f(1)= 2

Since f(x) is continuous and f(0) –1 and f(1) = 2, by the IVT there exists a value c on the given interval such that f(c) = 0. Therefore, x = c would be a solution of the given equation.