**In Exercises 55–64, find the particular solution that satisfies the initial condition.**

| *Differential Equation* | *Initial Condition* |
|---|---|
| 55. $yy' - e^x = 0$ | $y(0) = 4$ |
| 56. $\sqrt{x} + \sqrt{y}y' = 0$ | $y(1) = 4$ |
| 57. $y(x + 1) + y' = 0$ | $y(-2) = 1$ |
| 58. $2xy' - \ln x^2 = 0$ | $y(1) = 2$ |
| 59. $y(1 + x^2)y' - x(1 + y^2) = 0$ | $y(0) = \sqrt{3}$ |
| 60. $y\sqrt{1 - x^2}\,y' - x\sqrt{1 - y^2} = 0$ | $y(0) = 1$ |
| 61. $\frac{du}{dv} = uv \sin v^2$ | $u(0) = 1$ |
| 62. $\frac{dr}{ds} = e^{r-2s}$ | $r(0) = 0$ |
| 63. $dP - kP\,dt = 0$ | $P(0) = P_0$ |
| 64. $dT + k(T - 70)\,dt = 0$ | $T(0) = 140$ |

**In Exercises 65 and 66, find an equation of the graph that passes through the point and has the indicated slope.**

| *Point* | *Slope* |
|---|---|
| 65. $(1, 1)$ | $y' = -\frac{9x}{16y}$ |
| 66. $(8, 2)$ | $y' = \frac{2y}{3x}$ |

**In Exercises 67 and 68, find all functions $f$ having the indicated property.**

67. The tangent to the graph of $f$ at the point $(x, y)$ intersects the $x$-axis at $(x + 2, 0)$.

68. All tangents to the graph of $f$ pass through the origin.

**In Exercises 69–76, determine whether the function is homogeneous, and if it is, determine its degree.**

69. $f(x, y) = x^3 - 4xy^2 + y^3$

70. $f(x, y) = x^3 + 3x^2y^2 - 2y^2$

71. $f(x, y) = \frac{x^2y^2}{\sqrt{x^2 + y^2}}$

72. $f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}}$

73. $f(x, y) = 2 \ln xy$

74. $f(x, y) = \tan(x + y)$

75. $f(x, y) = 2 \ln \frac{x}{y}$

76. $f(x, y) = \tan \frac{y}{x}$

**In Exercises 77–82, solve the homogeneous differential equation.**

77. $y' = \frac{x + y}{2x}$

78. $y' = \frac{x^3 + y^3}{xy^2}$

79. $y' = \frac{x - y}{x + y}$

80. $y' = \frac{x^2 + y^2}{2xy}$

81. $y' = \frac{xy}{x^2 - y^2}$

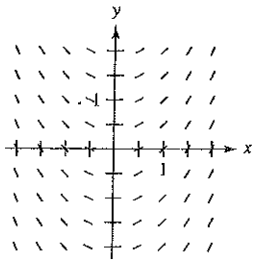
82. $y' = \frac{2x + 3y}{x}$

**In Exercises 83–86, find the particular solution that satisfies the initial condition.**

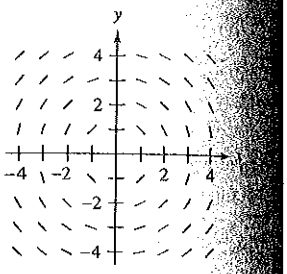
| *Differential Equation* | *Initial Condition* |
|---|---|
| 83. $x\,dy - (2xe^{-y/x} + y)\,dx = 0$ | $y(1) = 0$ |
| 84. $-y^2\,dx + x(x + y)\,dy = 0$ | $y(1) = 1$ |
| 85. $\left(x \sec \frac{y}{x} + y\right)dx - x\,dy = 0$ | $y(1) = 0$ |
| 86. $(2x^2 + y^2)\,dx + xy\,dy = 0$ | $y(1) = 0$ |

*Slope Fields* **In Exercises 87–90, sketch a few solutions of the differential equation on the slope field and then find the general solution analytically. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

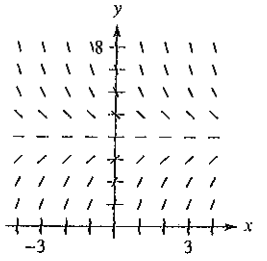
87. $\frac{dy}{dx} = x$



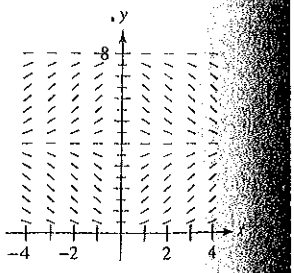

88. $\frac{dy}{dx} = -\frac{x}{y}$




89. $\frac{dy}{dx} = 4 - y$




90. $\frac{dy}{dx} = 0.25x(4 - y)$




**In Exercises 91–94, use a computer algebra system to sketch the slope field for the differential equation, and graph the solution satisfying the specified initial condition.**

91. $\frac{dy}{dx} = 0.5y,\ y(0) = 6$

92. $\frac{dy}{dx} = 2 - y,\ y(0) = 4$

93. $\frac{dy}{dx} = 0.02y(10 - y),\ y(0) = 2$

94. $\frac{dy}{dx} = 0.2x(2 - y),\ y(0) = 9$

95. ***Radioactive Decay*** The rate of decomposition of radioactive radium is proportional to the amount present at any time. The half-life of radioactive radium is 1620 years. What percent of a present amount will remain after 25 years?

96. ***Chemical Reaction*** In a chemical reaction, a certain compound changes into another compound at a rate proportional to the unchanged amount. If initially there is 20 grams of the original compound, and there is 16 grams after 1 hour, when will 75 percent of the compound be changed?

Larson 7th ed.

Annual rate: 9.50%

Time to double: 7.30 years

$112,087.09 **51.** $30,688.87

(a) 10.24 years **55.** (a) 8.50 years

(b) 9.93 years (b) 8.18 years

(c) 9.90 years (c) 8.16 years

(d) 9.90 years (d) 8.15 years

7.43 million **59.** 6.83 million

When $k > 0$, the population is increasing.

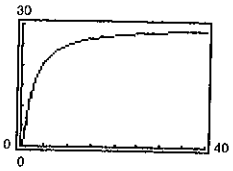
When $k < 0$, the population is decreasing.

527.06 millimeters of mercury

(a) $N \approx 30(1 - e^{-0.0502t})$ (b) 36 days

(a) $S \approx 30e^{-1.7918/t}$ (b) 20,965 units

(c)




2014 ($t = 16$)

a) 20 decibels (b) 70 decibels

c) 95 decibels (d) 120 decibels

a) $10^{8.3} \approx 199{,}526{,}231.5$ (b) $10^R$ (c) $\dfrac{1}{I \ln 10}$

False. The rate of growth $\dfrac{dy}{dx}$ is proportional to $y$.

True

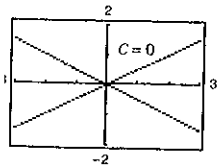
## *ion 5.7 (page 377)*

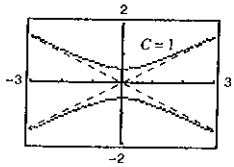
roof **3.** Proof **5.** Proof **7.** Not a solution

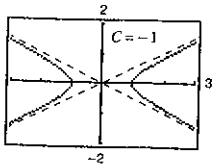
olution **11.** Solution **13.** Not a solution

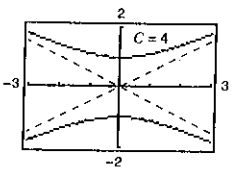
olution **17.** Not a solution **19.** $k = 0.07$

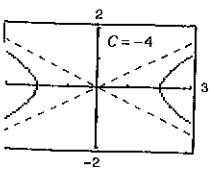
$y^2 = x^3$




**25.** $y = 3e^{-2x}$ **27.** $y = 2 \sin 3x - \frac{1}{3} \cos 3x$

**29.** $y = -2x + \frac{1}{2}x^3$ **31.** $y = x^3 + C$

**33.** $y = \frac{1}{2} \ln(1 + x^2) + C$

**35.** $y = x - \ln x^2 + C$ **37.** $y = -\frac{1}{2} \cos 2x + C$

**39.** $y = \frac{2}{5}(x - 3)^{5/2} + 2(x - 3)^{3/2} + C$ **41.** $y = \frac{1}{2}e^{x^2} + C$

**43.** $y^2 - x^2 = C$ **45.** $r = Ce^{0.05s}$ **47.** $y = C(x + 2)^3$

**49.** $y^2 = C - 2 \cos x$ **51.** $y = -\frac{1}{4}\sqrt{1 - 4x^2} + C$

**53.** $y = Ce^{(\ln x)^2/2}$ **55.** $y^2 = 2e^x + 14$ **57.** $y = e^{-(x^2+2x)/2}$

**59.** $y^2 = 4x^2 + 3$ **61.** $u = e^{(1-\cos v^2)/2}$ **63.** $P = P_0e^{kt}$

**65.** $9x^2 + 16y^2 = 25$ **67.** $f(x) = Ce^{-x/2}$

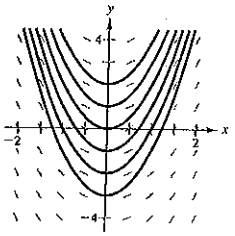
**69.** Homogeneous of degree 3 **71.** Homogeneous of degree 3

**73.** Not homogeneous **75.** Homogeneous of degree 0

**77.** $|x| = C(x - y)^2$ **79.** $|y^2 + 2xy - x^2| = C$

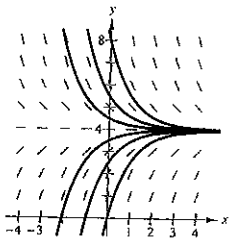
**81.** $y = Ce^{-x^2/2y^2}$ **83.** $e^{y/x} = 1 + \ln x^2$ **85.** $x = e^{\sin(y/x)}$

**87.**



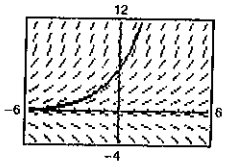

$y = \frac{1}{2}x^2 + C$

**89.**

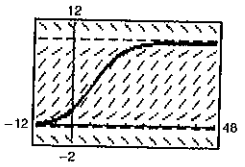



$y = 4 + Ce^{-x}$

**91.**




**93.**




**95.** 98.9% of the original amount

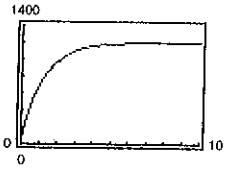
**97.** (a) $\dfrac{dy}{dx} = k(y - 4)$ (b) a (c) Proof

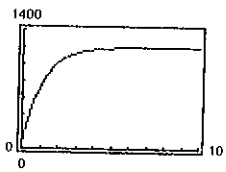
**98.** (a) $\dfrac{dy}{dx} = k(x - 4)$ (b) b (c) Proof

**99.** (a) $\dfrac{dy}{dx} = ky(y - 4)$ (b) c (c) Proof

**100.** (a) $\dfrac{dy}{dx} = ky^2$ (b) d (c) Proof

**101.** (a) $w = 1200 - 1140e^{-0.8t}$ $w = 1200 - 1140e^{-0.9t}$




$w = 1200 - 1140e^{-t}$