Answers to Day 5 Homework

1. $\dfrac{dy}{dx} = \dfrac{3\cos(3t)}{2e^{2t}}$

2. Length $= \int_0^{\frac{\pi}{2}} \sqrt{9\cos^4 t \sin^2 t + 4\sin^2 t \cos^2 t}\, dt$

3. $\dfrac{dy}{dx} = \dfrac{4t^3 + 4t - 8}{3t^2 - 2t}$ is undefined when $3t^2 - 2t = 0$.

   So the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ has a vertical tangent when $t = 0$ and $t = \dfrac{2}{3}$.

4. $v(t) = \left\langle 2t, \dfrac{2}{2t+3} \right\rangle$, $a(t) = \left(2, -\dfrac{4}{(2t+3)^2} \right)$

5. $\left.\dfrac{dy}{dx}\right|_{t=1} = \left.\dfrac{3t^2 - 4}{6t - 1}\right|_{t=1} = -\dfrac{1}{2}$. When $t = 1$, $x = 1$, $y = -3$.

   Tangent line equation: $y + 3 = -\dfrac{1}{2}(x - 1)$ $\quad y - y = m(x - x)$

6. $e^t = x - 1$ so $\cancel{e^{2t} = x^2 - 2x + 1}$. Then $y = 2e^{2t}$ $\quad y = 2(e^t)^2$ $\quad$ so $y = 2x^2 - 4x + 2$.

7. Speed $= \left.\sqrt{(-5\sin(5t))^2 + (3t^2)^2}\right|_{t=2} = 12.304$ $\quad \boxed{y = 2(x-1)^2}$

8. **(a)** Magnitude $= \left.\sqrt{(t-2)^4+(2t-4)^2}\right|_{t=1} = \sqrt{5}$

   (b) Distance $= \int_0^1 \sqrt{(t-2)^4+(2t-4)^2}\,dt = 3.816$

   (c) The particle is at rest when $v(t) = \left\langle (t-2)^2, 2t-4 \right\rangle = \langle 0,0 \rangle$, so is at rest when $t = 2$. Position = (4, 0)

9. $a(5) = \langle 10.178,\ 6.277 \rangle$, speed $= \left.\sqrt{\left(1+\tan\left(t^2\right)\right)^2+\left(3e^{\sqrt{t}}\right)^2}\right|_{t=5} = 28.083$

10. $3t + 2\sin t = 5$ when $t = 1.079...$
    $v(1.079...) = \langle 0.119,\ 3.944 \rangle$

11. (a) $\dfrac{dy}{dx} = \left.\dfrac{\cos\left(t^2\right)}{2\sin\left(t^3\right)}\right|_{t=1} = 0.321$

    Tangent line equation: $y - 4 = 0.321(x-3)$

    (b) Speed $= \left.\sqrt{4\sin^2\left(t^3\right)+\cos^2\left(t^2\right)}\right|_{t=2} = 2.084$

    (c) Distance $= \int_0^1 \sqrt{4\sin^2\left(t^3\right)+\cos^2\left(t^2\right)}\,dt = 1.126$

    (d) $x(2) = \overset{x(1)}{\underline{3}} + \int_1^2 2\sin\left(t^3\right)dt = 3.436,\ y(2) = 4 + \int_1^2 \cos\left(t^2\right)dt = 3.557$ so position = (3.436, 3.557)

$\int_1^2 v(t)\,dt \overset{?}{=} x(2) - \underline{x(1)} = x(2) - 3$

$\int_1^2 v(t)\,dt + 3 = x(2)$