Day 4 HW

① $\frac{dy}{dx} = \frac{e^{t^3} \cdot 3t^2}{2t} = \frac{3te^{t^3}}{2}$

② $\vec{v} = \left\langle \frac{1(2t+5)}{(t^2+5t)}, 6t \right\rangle \quad \vec{v}(2) = \langle 9/14, 12 \rangle$

③ $\vec{v} = \langle 5t^4, 12t^3 - 6t^2 \rangle \quad \vec{a}(1) = \langle 20, 24 \rangle$

$\vec{a} = \langle 20t^3, 36t^2 - 12t \rangle$

④ $\vec{v} = \langle 3\cos(3t - \pi/2), 6t \rangle \quad \vec{v}\left(\frac{\pi}{2}\right) = \left\langle 3\cos\left(\frac{3\pi}{2} - \frac{\pi}{2}\right), 6\pi/2 \right\rangle$

$= \langle -3, 3\pi \rangle$

⑤ $y = \ln x \quad x'(t) = t+1$

$x = \frac{1}{2}t^2 + t + C$

$1 = 0 + 0 + C$

$1 = C$

$x = \frac{1}{2}t^2 + t + 1 \qquad x(1) = \frac{1}{2} + 1 + 1 = 2.5$

$y = \ln\left(\frac{1}{2}t^2 + t + 1\right) \qquad y(1) = \ln(2.5)$

⑥ $\vec{v} = \langle 1+t, t^3 \rangle \quad \vec{s}(0) = \langle 5, 0 \rangle \quad \vec{s}(2) = ?$

$\vec{s} = \langle t + \frac{1}{2}t^2 + C_1, \frac{1}{4}t^4 + C_2 \rangle \qquad \vec{s}(2) = \langle 2+2+5, \frac{1}{4}(16) \rangle$

$\langle 5, 0 \rangle = \langle C_1, C_2 \rangle \qquad = \langle 9, 4 \rangle$

$\vec{s} = \langle \frac{1}{2}t^2 + t + 5, \frac{1}{4}t^4 \rangle$

(7) $xy = 10 \quad x = 2 \quad \frac{dy}{dt} = 3 \quad \frac{dx}{dt} =$

$10x^{-1} = y = \frac{10}{x} \quad y = 5$

$y' = -10x^{-2}$

$y' = \frac{-10}{x^2} = \frac{dy/dt}{dx/dt} = \frac{3}{dx/dt} \qquad \frac{dx}{dt} = \frac{3x^2}{-10} = \frac{3(2)^2}{-10} = \frac{12}{-10}$

$= -4/5$

(8) $x' = 3t^2 - 3t - 18 \quad y' = 3t^2 - 12t + 9$

$0 = 3(t^2 - t - 6) \qquad 0 = 3(t^2 - 4t + 3)$

$0 = (t+2)(t-3) \qquad 0 = (t-1)(t-3)$

$t = -2, 3 \qquad t = 1, 3$

at rest $t = 3$

(9) $x = t^3 \quad y = t^2 - 5t + 2 \qquad 8 = t^3$

$x' = 3t^2 \quad y' = 2t - 5 \qquad 2 = t$

$\frac{dy}{dx} = \frac{2t-5}{3t^2}\Big|_{t=2} = \frac{-1}{12} \qquad y + 4 = -\frac{1}{12}(x - 8)$

(10) $25 = 5t + 3\sin t \qquad \vec{v} = \langle 5 + 3\cos t, -1 + 8\sin t + \cos t - t\sin t \rangle$

$t = 5.4457552 \qquad \vec{v}(5.4457552) = \langle 7.008, -2.229 \rangle$

(11) a) $\sqrt{(2t)^2 + (2t^2)^2}$

$|\vec{v}(5)| = \sqrt{10^2 + 50^2} = \sqrt{100 + 2500} = \sqrt{2600} = 10\sqrt{26}$

b) $\int_0^5 |\vec{v}|\, dt = \int_0^5 \sqrt{4t^2 + 4t^4}\, dt = \int_0^5 \sqrt{4t^2(1+t^2)}\, dt$

$= \int_0^5 \overset{du}{2t}(1 + \overset{u}{t^2})^{1/2} dt = \frac{2}{3}(1+t^2)^{3/2}\Big|_0^5$

$= \frac{2}{3}(26)^{3/2} - 2/3$

c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t$ $\quad x = t^2 - 3$

$x + 3 = t^2$

$\frac{dy}{dx} = \sqrt{x+3}$ $\quad \sqrt{x+3} = t$

(12) $\frac{dx}{dt} = \frac{1}{t+1}$ $\quad \frac{dy}{dt} = 2t$ $\quad t \geq 0$

a) $x = \ln(t+1) + C_1$ $\quad y = t^2 + C_2$ $\quad P(x,y) = \langle \ln(t+1), t^2 - 1 \rangle$

$\ln 2 = \ln(1+1) + C_1$ $\quad 0 = 1^2 + C_2$

$0 = C_1$ $\quad -1 = C_2$

b) $e^{x} = e^{\ln(t+1)}$ $\quad y = t^2 - 1$

$y = (e^x - 1)^2 - 1$

$e^x = t+1$ $\quad y = e^{2x} - 2e^x$

$e^x - 1 = t$

c) $t = 0$ $\quad t = 4$

$x = \ln(1) = 0$ $\quad x = \ln 5$

$y = -1$ $\quad y = 15$

average rate of change $\frac{y-y}{x-x} = \frac{16}{\ln 5}$

d) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{1/(t+1)}\Big|_{t=1} = \frac{2}{1/2} = 4$

(13) a) $\frac{dy}{dx} = \frac{2\cos t}{3\sin t} = \frac{2}{3}\cot t \Big|_{\pi/4} = \frac{2}{3}$

b) $y = 3 + 2(\frac{\sqrt{2}}{2}) = 3 + \sqrt{2}$

$x = 2 - 3(\frac{\sqrt{2}}{2}) = 2 - \frac{3\sqrt{2}}{2}$

$y - (3+\sqrt{2}) = \frac{2}{3}\left[x - \left(2 - \frac{3\sqrt{2}}{2}\right)\right]$

(c) $0 = 2 - 3\cos t$ $\quad t = {}^{b}.8410686706$

$3\cos t = 2$ $\quad {}^{a}\, -.8410686706$

$\cos t = \frac{2}{3}$

$L = \int_a^b \sqrt{(3\sin t)^2 + (2\cos t)^2}$

$= 3.7566$