

Day 4 Homework

Use your calculator on problems 10 and 13c only.

- If $x = t^2 - 1$ and $y = e^{t^3}$, find $\frac{dy}{dx}$.
- If a particle moves in the xy -plane so that at any time $t > 0$, its position vector is $\langle \ln(t^2 + 5t), 3t^2 \rangle$, find its velocity vector at time $t = 2$.
- A particle moves in the xy -plane so that at any time t , its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.
- If a particle moves in the xy -plane so that at time t its position vector is $\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \rangle$, find the velocity vector at time $t = \frac{\pi}{2}$.
- A particle moves on the curve $y = \ln x$ so that its x -component has derivative $x'(t) = t + 1$ for $t \geq 0$. At time $t = 0$, the particle is at the point $(1, 0)$. Find the position of the particle at time $t = 1$.
- A particle moves in the xy -plane in such a way that its velocity vector is $\langle 1 + t, t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5, 0 \rangle$, find the position of the particle at $t = 2$.
- A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$,
what is the value of $\frac{dx}{dt}$?
- The position of a particle moving in the xy -plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?
- A curve C is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of C at the point $(8, -4)$.
- A particle moves in the xy -plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$. Find the velocity vector at the time when the particle's horizontal position is $x = 25$.
- The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.
 - Find the magnitude of the velocity vector at time $t = 5$.
 - Find the total distance traveled by the particle from $t = 0$ to $t = 5$.
 - Find $\frac{dy}{dx}$ as a function of x .

12. Point $P(x, y)$ moves in the xy -plane in such a way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$.

(a) Find the coordinates of P in terms of t given that, when $t = 1$, $x = \ln 2$ and $y = 0$.

(b) Write an equation expressing y in terms of x .

(c) Find the average rate of change of y with respect to x as t varies from 0 to 4.

(d) Find the instantaneous rate of change of y with respect to x when $t = 1$.

13. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

(a) Find $\frac{dy}{dx}$ as a function of t .

(b) Find the equation of the tangent line at the point where $t = \frac{\pi}{4}$.

(c) The curve C intersects the y -axis twice. Approximate the length of the curve between the two y -intercepts.