Answers to Day 6 Homework

1. (a) Magnitude $= \sqrt{(2t)^2 + (2t^2)^2}\Big|_{t=2} = 4\sqrt{5}$

   (b) Distance $= \int_0^4 \sqrt{4t^2 + 4t^4}\, dt$

   (c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t = \sqrt{x+2}$

   (d) Particle is on the y-axis when $t = \sqrt{2}$, and $\frac{dx}{dt} = 2(t-1)$

   $\vec{a}(\sqrt{2}) =$
   $\langle 2, 4\sqrt{2} \rangle$

2. (a) $x(t) = \int \frac{1}{t+1}\, dt = \ln(t+1) + C$. Since $x(1) = \ln 2$, $C = 0$.

   $y(1) = 4,\ D = 3$. Since $y(1) = 4,\ D = 3$.

   Position vector $= \left(\ln(t+1), t^2 + 3\right)$

   (b) When $t = 1$, $\frac{dy}{dx} = \frac{2}{\frac{1}{2}} = 4$ so the tangent line equation is $y - 4 = 4(x - \ln 2)$.

   (c) Magnitude $= \sqrt{\left(\frac{1}{t+1}\right)^2 + (2t)^2}\Bigg|_{t=1} = \frac{\sqrt{17}}{2}$

   (d) $\frac{dy}{dx} = \frac{2t}{\frac{1}{t+1}} = 2t(t+1) = 12$ when $2t^2 + 2t - 12 = 0$ so $t = 2$

3. $t^2 + 2\cos t = 7$ when $t = 2.996\ldots$ $\quad v(2.996\ldots) = \langle -0.968,\ 5.704 \rangle$

4. $\frac{dy}{dt} = \left(\frac{dx}{dt}\right)(t+3) = \left(1 + \sin t^3\right)(t+3)$ so $a(2) = \langle -1.746,\ -6.741 \rangle$

5. (a) When $t = 1$,

   $\frac{dy}{dx} = \frac{\sin(e^t)}{\cos(e^t)}\Bigg|_{t=1} = -0.451$ so the tangent line equation is $y - 2 = -0.451(x - 3)$

   (b) Speed $= \sqrt{\left(\cos(e^t)\right)^2 + \left(\sin(e^t)\right)^2}\Big|_{t=1} = 1$

   (c) Distance $= \int_0^2 \sqrt{\left(\cos(e^t)\right)^2 + \left(\sin(e^t)\right)^2}\, dt = 2$

   (d) $x(2) = 3 + \int_1^2 \cos(e^t)\, dt = 2.896$, $y(2) = 2 + \int_1^2 \sin(e^t)\, dt = 1.676$

   so position = (2.896, 1.676)

$\int_1^2 v(t) =$ position

$\int_1^2 v(t) = x(2) - x(1)$

-3

6. (a) $a(3)=\langle 11.029,\ 23.545\rangle$

(b)
$$\frac{dy}{dx}=\left.\frac{\cos\left(t^3-t\right)}{\sin\left(t^3-t\right)}\right|_{t=3}=-0.468$$ so the tangent line equation is $y-4=-0.468(x-1)$

(c) Magnitude $=\left.\sqrt{\left(\sin\left(t^3-t\right)\right)^2+\left(\cos\left(t^3-t\right)\right)^2}\right|_{t=3}=1$

(d) $x(2)=1-\int_2^3\sin\left(t^3-t\right)dt=0.932,\ y(2)=4-\int_2^3\cos\left(t^3-t\right)dt=4.002$ so the position = (0.932, 4.002)

7. (a) $y(1)=5-\int_1^3\left(2+\sin\left(e^t\right)\right)dt=1.269$

(b) $\frac{dy}{dx}=\frac{2+\sin\left(e^t\right)}{\frac{dx}{dt}}=-1.8$ so $\left.\frac{dx}{dt}\right|_{t=3}=\frac{2+\sin\left(e^3\right)}{-1.8}=-1.636$

(c) Speed $=\sqrt{\left(\frac{2+\sin\left(e^3\right)}{-1.8}\right)^2+\left(2+\sin\left(e^3\right)\right)^2}=3.368$