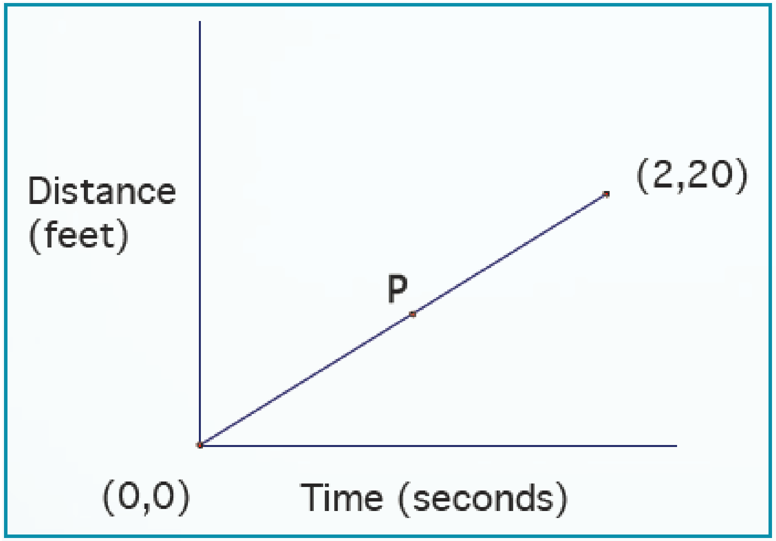
Chapter 2 Derivatives

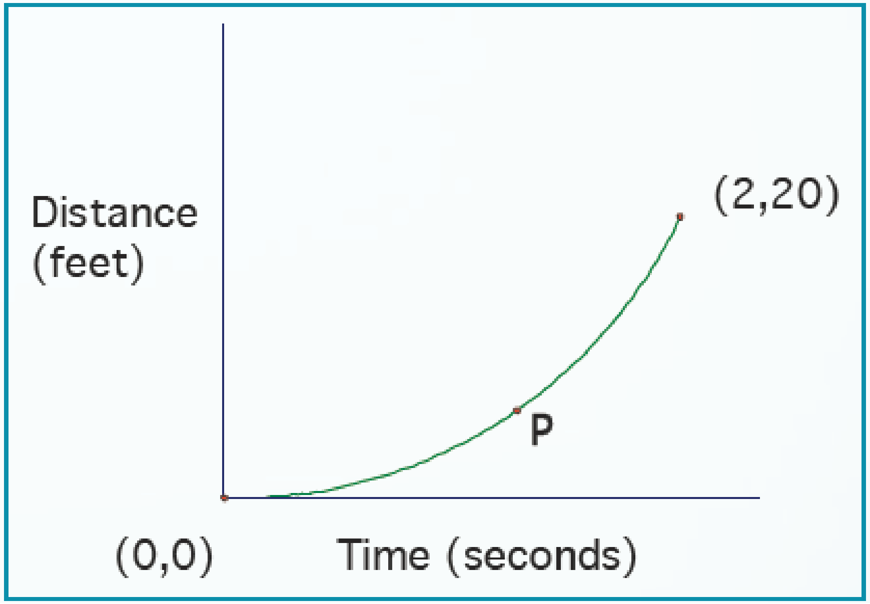


What is the slope of the line?

What is the average rate of change from t=0 to t=2?

What is the slope at point P?

What is the average rate of change at point P?



What is the average rate of change from t=0 to t=2?

What is the average slope?

What is the slope at point P?

What is the rate of change at point P?

Definitions

- **Average rate of change**: slope between two points
- **Instantaneous rate of change**: slope or rate of change at a specific point, also called the derivative at a point
- **Derivative**: formula used to calculate slopes/derivatives at specific points on a function

- For linear graphs, average and instantaneous rates of change are the same
- For all other graphs, average and instantaneous rates are not the same

- If you travel 100 miles in 2 hours, your average rate of change is 50 mph, but your instantaneous rate of change varies from moment to moment during the 2 hour period.

Who is the father of Calculus?

Newton vs. Leibniz

- There was much controversy over who (and thus which country) should be credited with calculus since both worked at the same time.
- Newton derived his results first, but Leibniz published first.

$$f'(x) \approx y' \approx \frac{dy}{dx} = \text{1st derivative}$$

$$f''(x) \approx y'' \approx \frac{d^2y}{dx^2} = \text{2nd derivative}$$

Notation

Given function $f(x)$

$f'(x) =$ derivative of function f

$f'(3) =$ derivative of function f at $x = 3$

$f''(x) =$ derivative of $f'(x)$

or the 2nd derivative of $f(x)$

$f'''(x) =$ derivative of $f''(x)$

or the 3rd derivative of $f(x)$

$f^{n}(x) =$ the nth derivative of f.

Given equation y

$y' =$ derivative of equation y

$y'(3) =$ derivative of y at $x = 3$

$y'' =$ derivative of y'

or the 2nd derivative of y

$y''' =$ derivative of y''

or the 3rd derivative of y

$y^{n} = nth$ derivative of y

Use of the prime marks is also known as Lagrange notation.

Leibniz Notation

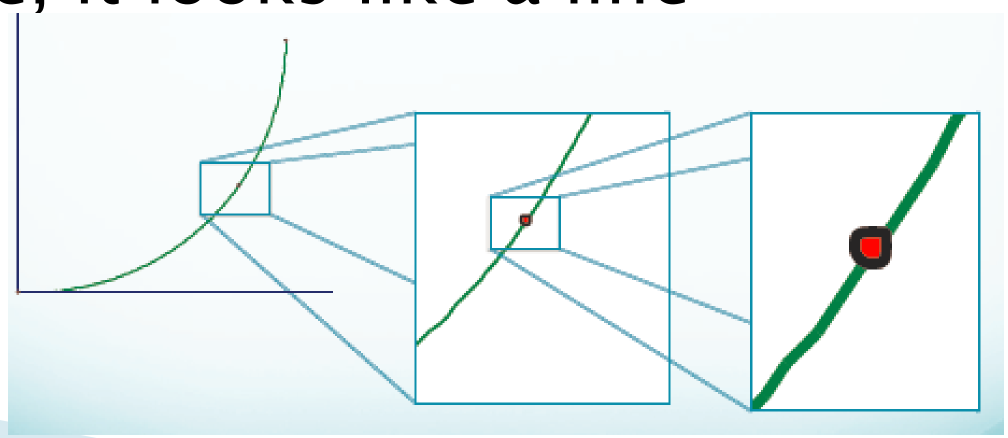
$$\frac{dy}{dx} = \text{derivative of } y \text{ with respect to } x$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} = \text{2nd derivative of } y$$

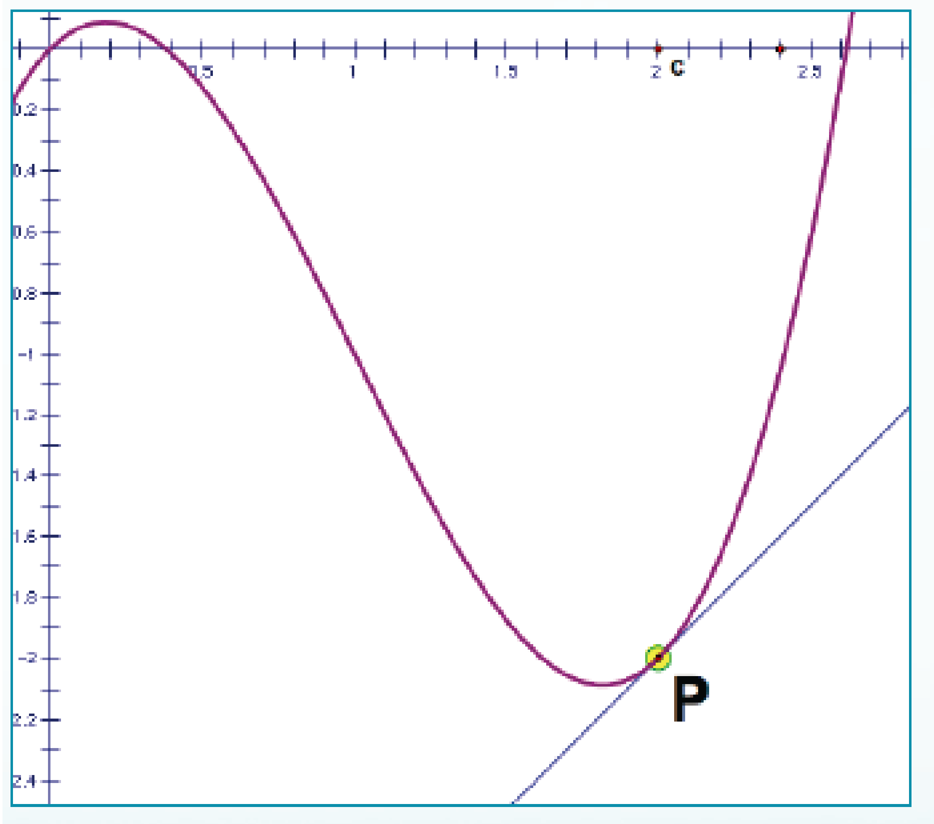
$$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3} = \text{3rd derivative of } y$$

How to find the derivative

- If graph is linear, use the slope formula
- If graph is curved, local linearization could be a way to approximate
- **Local Linearity**: at a point, a curve behaves indistinguishably from a straight line; when you zoom in a curve, it looks like a line

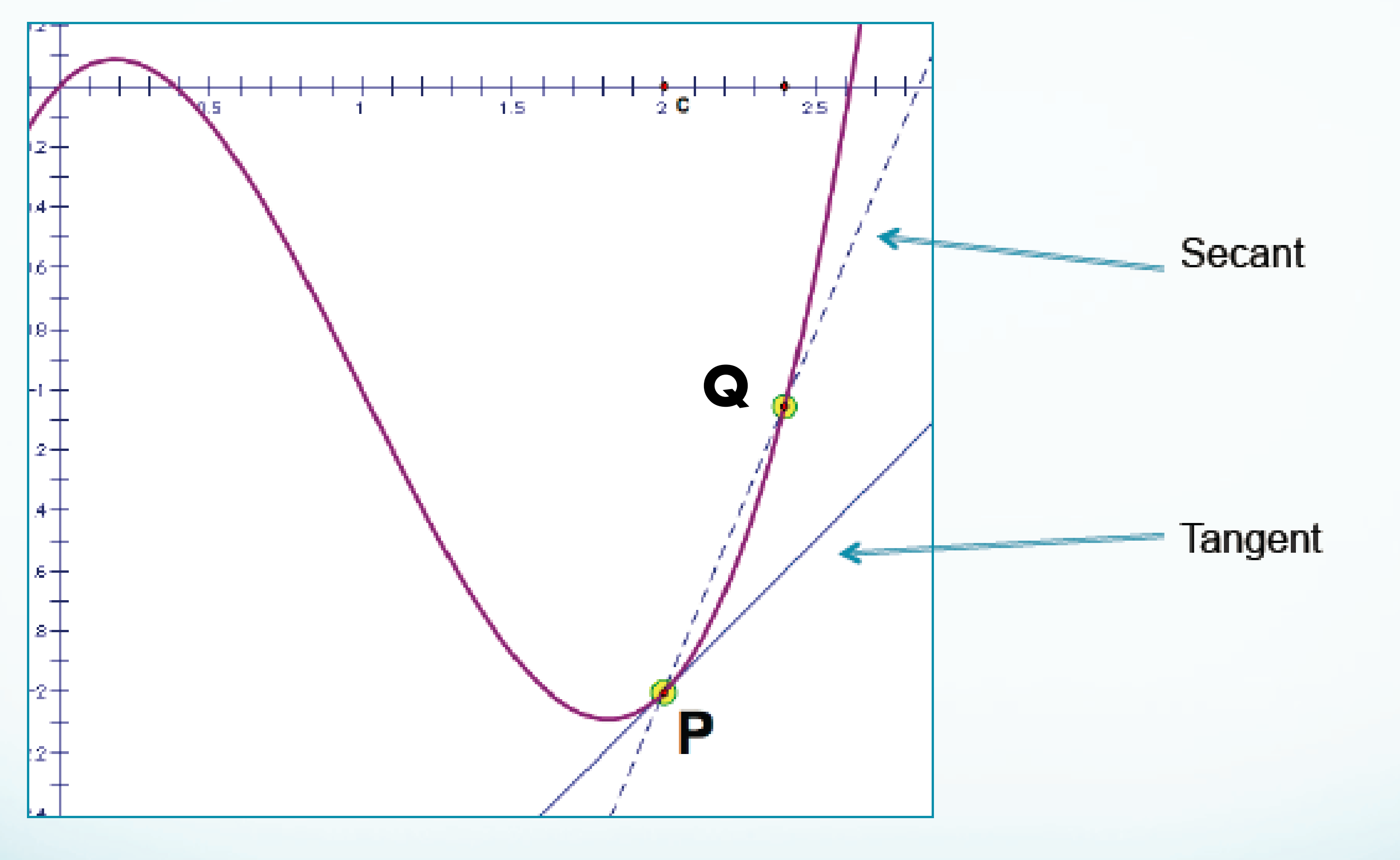


If we extend that zoomed-in linear piece of the curve, we get a tangent line at the point.

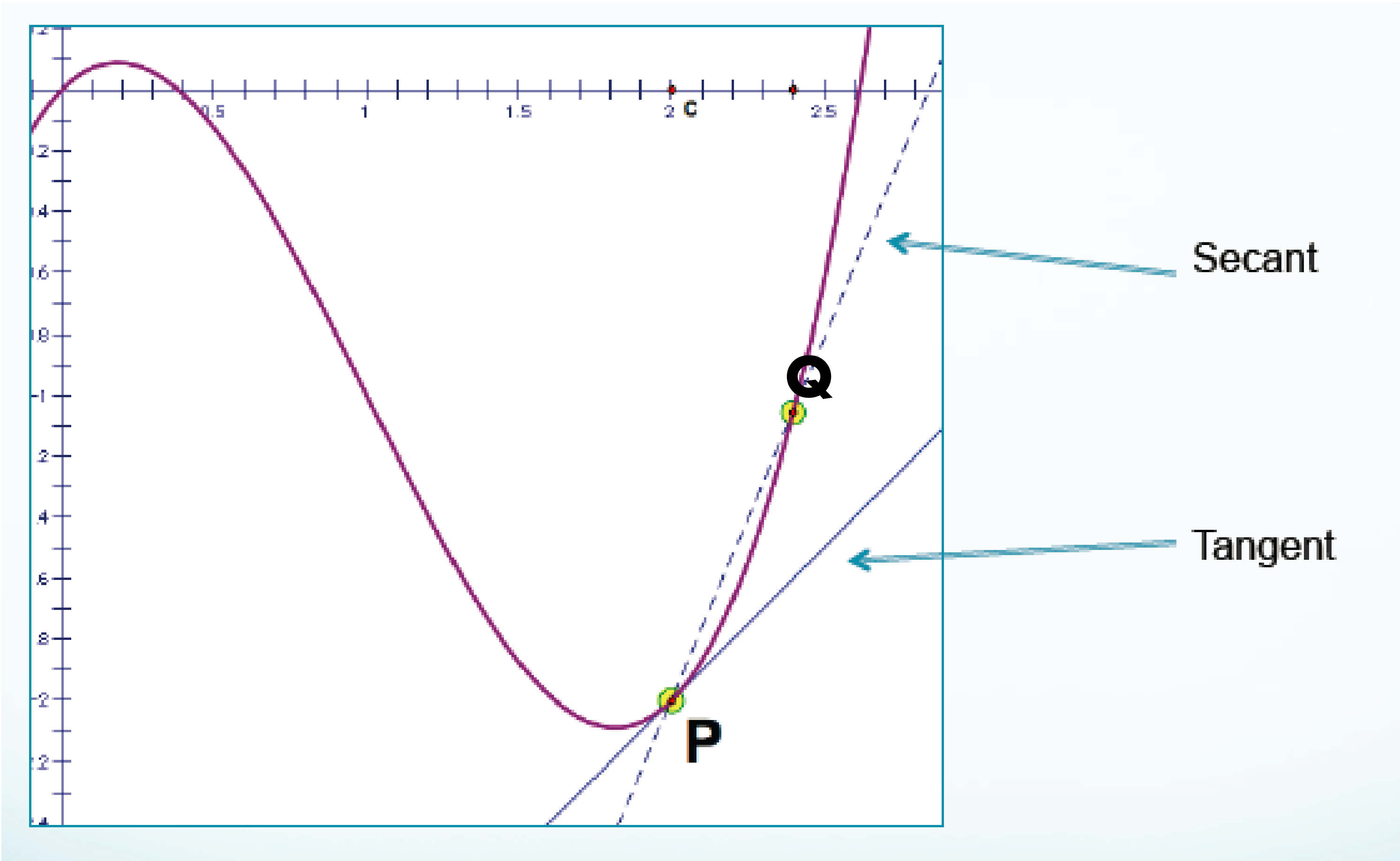


The derivative of a function at a point is the slope of the tangent line at that point.

We need two points to calculate slope. We only have one, P.



- Let’s pick a second point and draw a secant line and use its slope to estimate the tangent’s slope. (sketchpad demo)



As Q gets closer to P, the slope of the secant will approach the slope of the tangent line. (As seen on sketchpad demo)

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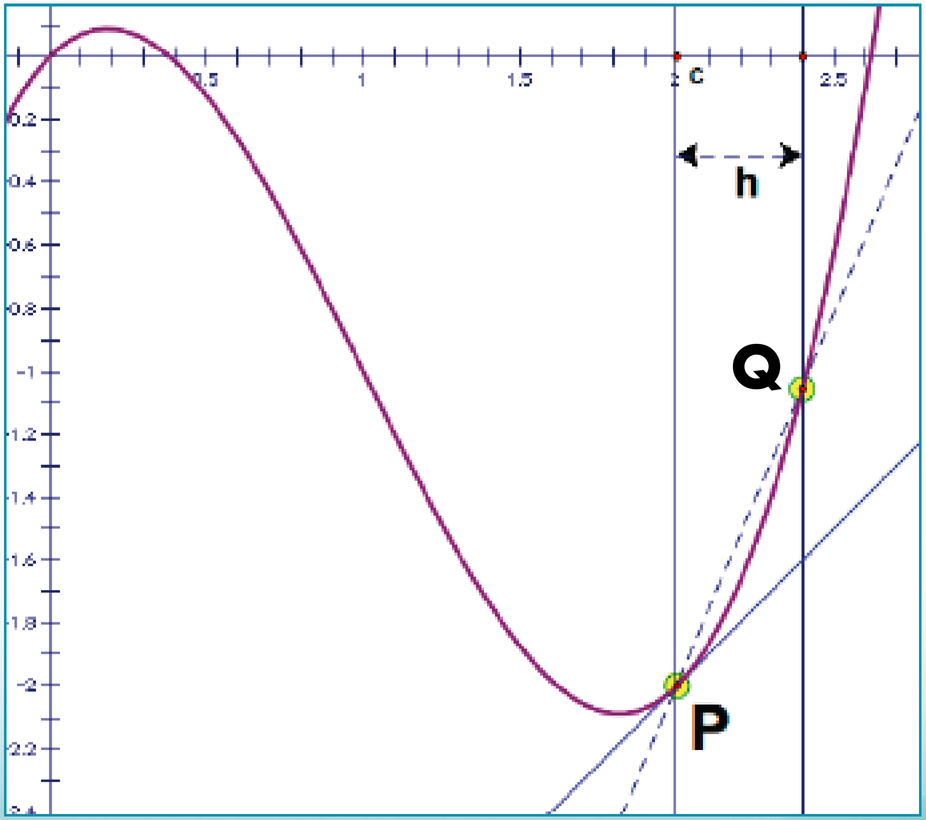


P(x, f(x))   Q(x + h, f(x + h))

Slope of secant $= \dfrac{f(x+h)-f(x)}{x+h-x}$

Slope of tangent =

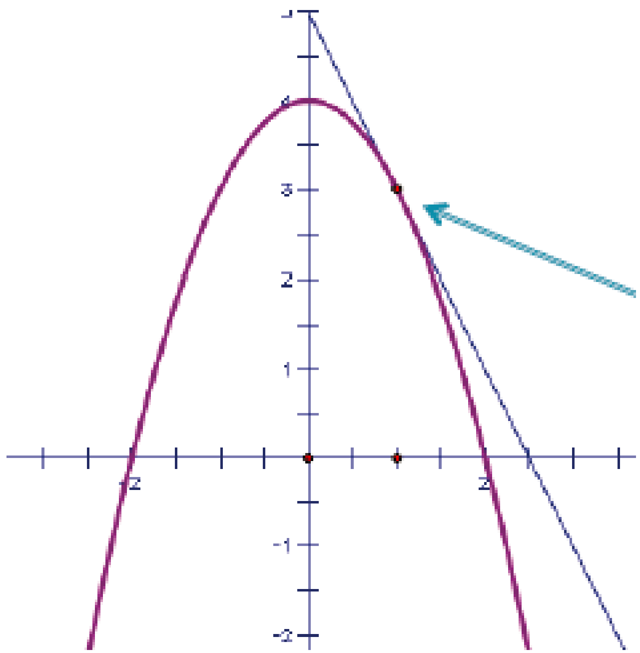
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{x+h-x} =$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)$$

$f(x) = 4 - x^2$ Find f'(1)

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

What does f'(1) = −2 mean?



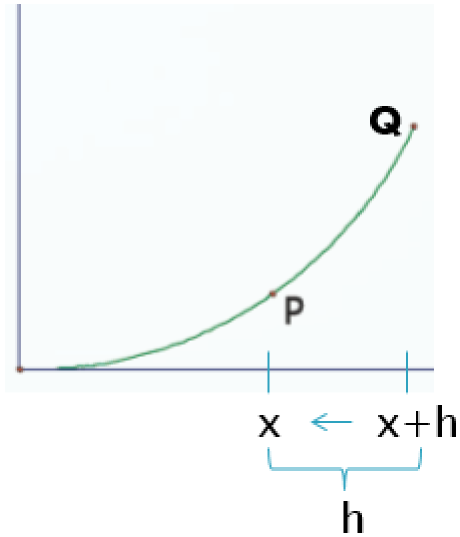
The slope of the tangent line at x = 1 is −2.

If you have a point and the slope, you can write the equation for the tangent line.

Or the equation of the line normal to the tangent line at x = 1.

You try: $y = \frac{3}{x}$

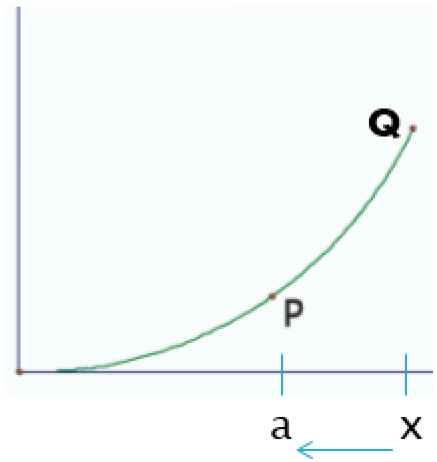
- Find a) y'(3)
- b) write the equation of the tangent line and normal line at x = 3



Definition #1:

Distance between points is approaching zero

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Definition #2: Point Q is approaching Point P

P(a, f(a))   Q (x, f(x))

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(x) = 4 - x^2$ Find f'(1)

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- If a function is <u>differentiable</u> (adjective) at a point, you can <u>differentiate</u> (verb) to find the <u>derivative</u> (noun).

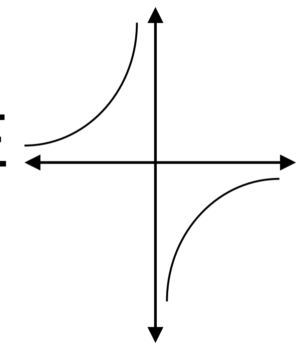
- To be differentiable at a point, the function must be continuous at that point.

- But being continuous at a point does not guarantee a function is differentiable at that point.

When derivatives fail to exist

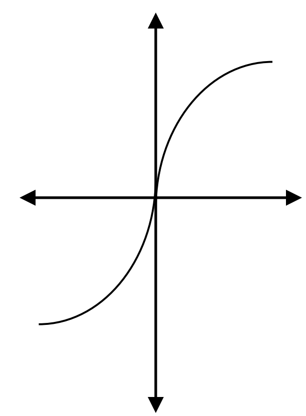
1. Discontinuous at a point

   If f(0) is undefined, then f'(0) DNE



2. Vertical tangent at a point

   Slope of a vertical line is undefined so the derivative does not exist



$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = +\infty$$

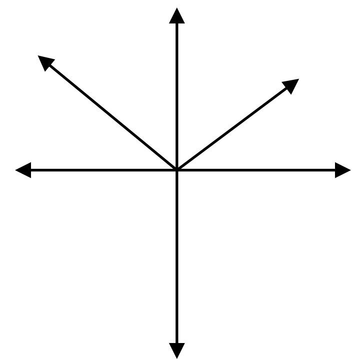
When derivatives fail to exist

- 3. Corner

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = 1$$

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = -1$$

Limits(slopes) do not agree at x = 0, so derivative does not exist

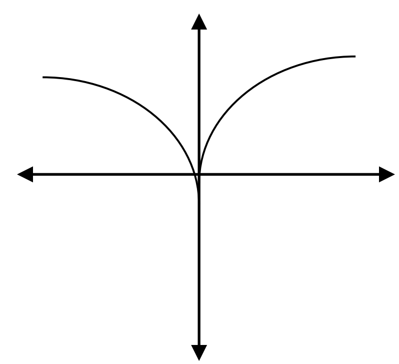


- 4. Cusp

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = -\infty$$

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = +\infty$$

Limits(slopes) do not agree at x = 0
And tangent is vertical, so slope is undefined



(Zoom in–does it look like a line?)