AP Calculus
Miss Brown

Name Answer Key
Date

Derivatives of Inverse Functions

Directions: Show your work neatly on a separate sheet of paper.

#7
$(g^{-1})'(3) = \frac{1}{g'(7)} = \frac{4}{3}$

#8
$g'(6) = \frac{1}{f'(2)} = \frac{1}{1/3} = 3$

$g(6) = 2$ b/c $f(2) = 6$

A. Find the derivative of f^{-1} for each of the following functions:

1. $f(x) = 5x^3 + x - 7$
   $x = 5y^3 + y - 7$
   $1 = 15y^2 \frac{dy}{dx} + \frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{15y^2+1}$
2. $f(x) = 2x^5 + x^3 + 1$
   $x = 2y^5 + y^3 + 1$
   $1 = 10y^4 \frac{dy}{dx} + 3y^2 \frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{10y^4+3y^2}$
3. $f(x) = 5x - \sin(2x)$
   $x = 5y - \sin(2y)$
   $1 = 5\frac{dy}{dx} - 2\cos(2y)\frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{5-2\cos(2y)}$

B. Evaluating the Derivatives of Inverse Functions

1. Find the derivative of the inverse of $f(x) = x^3 + 7x + 2$ at the point where $f^{-1}(10) = 1$.
   $x = y^3 + 7y + 2$
   $1 = 3y^2\frac{dy}{dx} + 7\frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{3y^2+7}$
   $\frac{dy}{dx} = \frac{1}{10}$
   $(f^{-1})'(10) = \frac{1}{f'(1)} = \frac{1}{10}$
   $f'(x) = 3x^2 + 7$ $f'(1) = 3+7 = 10$
2. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?
   $x = y^3 + y$
   $1 = 3y^2\frac{dy}{dx} + \frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{3y^2+1}$
   $\frac{dy}{dx} = \frac{1}{4}$
   $g'(2) = \frac{1}{f'(1)} = \frac{1}{4}$
   $f'(x) = 3x^2 + 1$ $f'(1) = 3+1 = 4$
3. Let f be the function defined by $f(x) = x^3 + 8x + \cos(3x)$. If $g(x) = f^{-1}(x)$ and $g(1) = 0$, find the value of $g'(1)$.
   $x = y^3 + 8y + \cos(3y)$
   $1 = 3y^2\frac{dy}{dx} + 8\frac{dy}{dx} - 3\sin(3y)\frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{3y^2+8-3\sin(3x)}$
   $g'(1) = \frac{1}{f'(0)} = \frac{1}{8}$
   $f'(x) = 3x^2 + 8 - 3\sin(3x)$ $f'(0) = 3(0)^2 + 8 - 3\sin 0 = 8$
   $\frac{dy}{dx} = \frac{1}{8}$
4. If $f(x) = x^5 + 3x + 2$ and $g(x) = f^{-1}(x)$, find $g'(2)$.
   $g'(2) = \frac{1}{f'(0)}$
   $x = y^5 + 3y + 2$
   $1 = 5y^4\frac{dy}{dx} + 3\frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{5y^4+3}$
   $\frac{dy}{dx} = \frac{1}{3}$
   $g(2) = ?$
   $f(?) = 2$
   $g(2) = 0$
   $2 = x^5 + 3x + 2$
   $0 = x^5 + 3x$
   $0 = x(x^4 + 3)$
   $x = 0$ never $= 0$
5. Find $(f^{-1})'(-1)$ if $f(x) = 3x - \cos x$.
   $(f^{-1})'(-1) = \frac{1}{f'(0)}$
   $x = 3y - \cos y$
   $1 = 3\frac{dy}{dx} + \sin y \frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{3+\sin y}$
   $\frac{dy}{dx} = \frac{1}{3}$
   $f^{-1}(-1) = ?$
   $f(?) = -1$
   $f^{-1}(-1) = 0$
   $3x - \cos x = -1$
   $0 - \cos 0 = -1$
   $-1 = -1$
6. Find $(f^{-1})'(5)$ if $f(x) = x^3 + 2x + 5$.
   $(f^{-1})'(5) = \frac{1}{f'(0)}$
   $x = y^3 + 2y + 5$
   $1 = 3y^2\frac{dy}{dx} + 2\frac{dy}{dx}$
   $\frac{dy}{dx} = \frac{1}{3y^2+2}$
   $\frac{dy}{dx} = \frac{1}{2}$
   $f^{-1}(5) = ?$
   $f(?) = 5$
   $f^{-1}(5) = 0$
   $x^3 + 2x + 5 = 5$
   $x^3 + 2x = 0$
   $x(x^2 + 2) = 0$
   $x = 0$ $x^2 + 2 \neq 0$