Derivative of an Inverse Function

SECTION 7.1

Find the inverse of $f(x)=\sqrt{2x-3}$

$$y=\sqrt{2x-3}$$

$$x=\sqrt{2y-3}$$

$$x^2=2y-3$$

$$x^2+3=2y$$

$$\frac{x^2+3}{2}=y$$

$$\frac{x^2+3}{2}=f^{-1}(x)$$

Domain of f(x)

$$\left[\frac{3}{2},\infty\right)$$

Range of f(x)

$$[0,\infty)$$

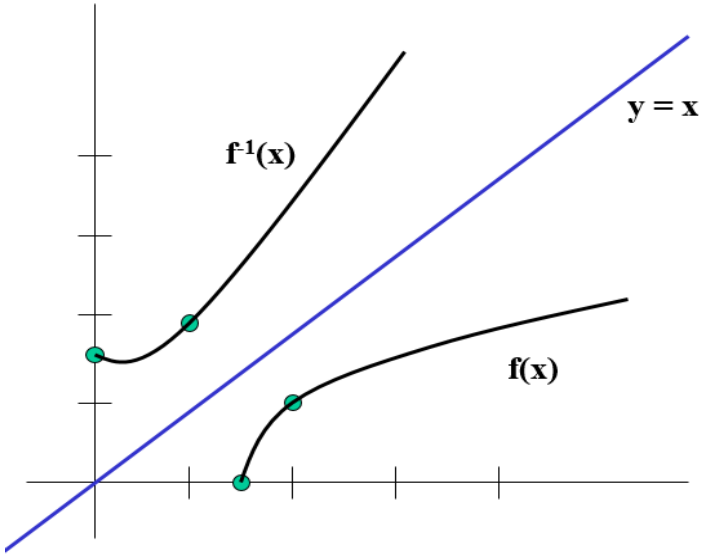
Domain of $f^{-1}(x)$ = Range of f(x)

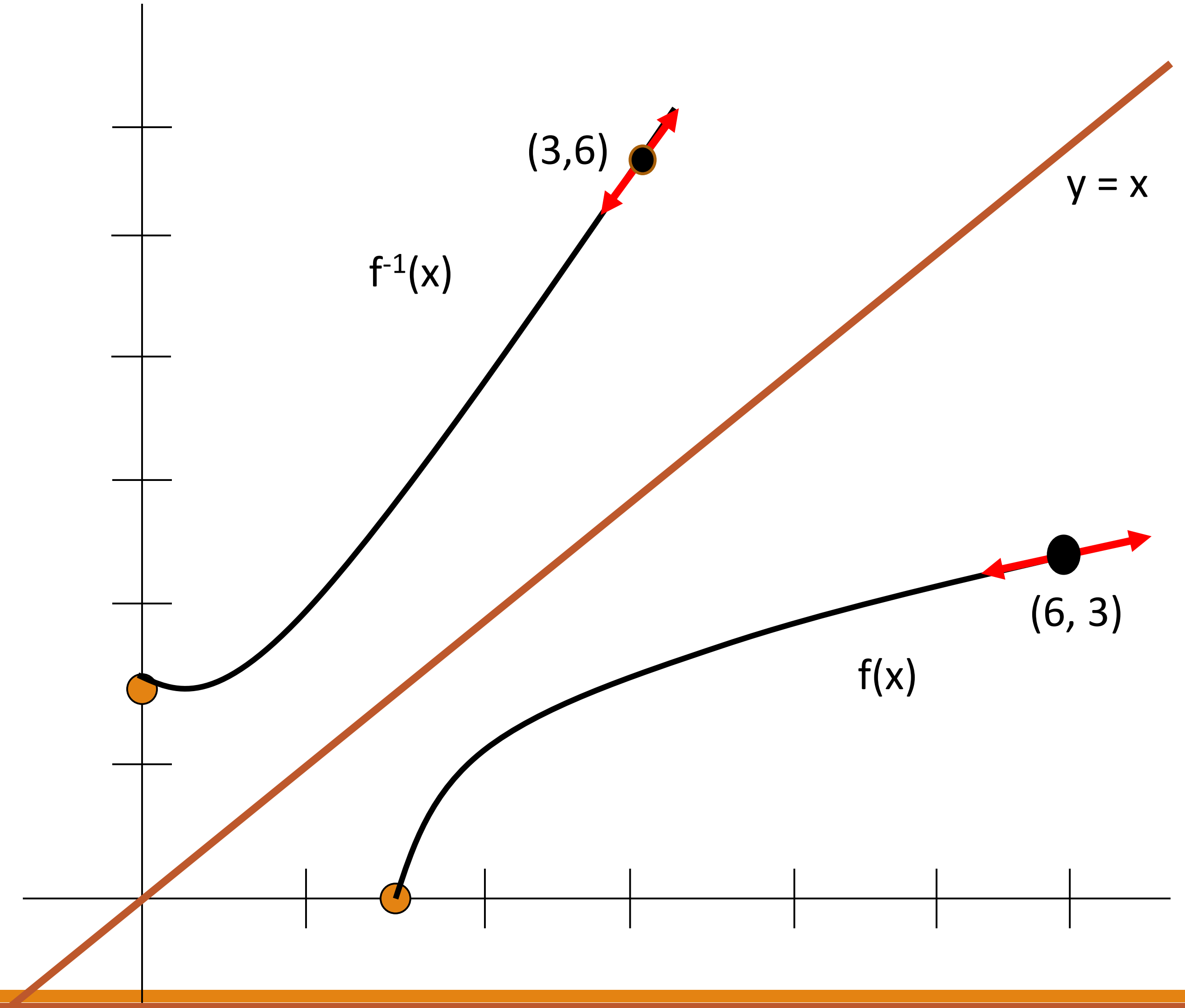
and

Range of $f^{-1}(x)$ = Domain of f(x)

Steps for finding an inverse.

1. exchange x's and y's
2. solve for y
3. replace y with f^{-1}




(3,6)
y = x
$f^{-1}(x)$
(6, 3)
f(x)

Find f'(6)

$$f(x)=\sqrt{2x-3}$$

$$f'(x) = \frac{1}{2}(2x-3)^{-1/2}\cdot 2 = \frac{1}{\sqrt{2x-3}}$$

$$f'(6) = \frac{1}{\sqrt{2\cdot 6-3}} = \frac{1}{3}$$

Find $(f^{-1})'(3)$

$$\frac{x^2+3}{2} = f^{-1}(x)$$

$$(f^{-1})'(x) = \frac{1}{2}(2x) = x$$

$$(f^{-1})'(3) = 3$$

If f is a one-to-one differentiable function with inverse function $g = f^{-1}$ and $f'(b) \neq 0$, then the inverse function is differentiable at (a,b) and

$$g'(a) = \frac{1}{f'(b)}$$

If f(x) = 2x + cos x, find $(f^{-1})'(1)$.

If $f(x) = \sqrt{x^3 + x^2 + x + 1}$, find $(f^{-1})'(2)$.

Try the following:

1. If $f(x) = x^5 - x^3 + 2x$, find $(f^{-1})'(1)$.

2. If $f(x) = 3 + x^2 + \tan\left(\frac{\pi}{2}x\right)$, $-1<x<1$, find $(f^{-1})'(1)$.

1. $\frac{1}{4}$

2. $\frac{2}{\pi}$