Derivatives and Integration of Series

Section 9.9 continued

Derivative of a Series

$$\frac{d}{dx}\left(3+3x+3x^2+3x^3\right)$$

Notice our derivative has one less term than the original series.

$$\frac{d}{dx}\left(3+3x+3x^2+3x^3+\cdots 3x^n+\cdots\right)$$

$$\frac{d}{dx}\sum_{n=0}^{\infty}3x^n=\sum_{n=1}^{\infty}3nx^{n-1}$$

Increased for lost
of 1st term whose
derivative is zero

Whatever is valid for a polynomial is usually good for a series.

Use Power and Chain rules.

n is a constant, x is a variable.

Interval and radius of convergence are the same.

$$\frac{d}{dx}\sum_{n=0}^{\infty} 4(2x)^n$$

The endpoints may or may not be included.

Integrals of Series

$$\int 3+3x+3x^2+3x^3 \; dx$$

$$\int 3+3x+3x^2+3x^3+\cdots+3x^n+\cdots \; dx$$

Find $\int \sum_{n=0}^{\infty} 5x^n dx$

$$\int \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

$$\int \sum_{n=0}^{\infty} \left(x^2\right)^n$$

Now let’s use derivatives and integrals to find series for functions not easily written as $\dfrac{a}{1-r}$.

$$f(x) = \frac{1}{(1-x)^2}$$

Express the function as a series

$$f(x) = \ln(1-x)$$

Express the function as a series

$$f(x) = \tan^{-1} x$$

Express the function as a series

$$f(x) = \frac{x^2}{(1+x)^2}$$

Series Manipulation Techniques

Substitute into a known Series

Yesterday we proved that $\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$

Find a series for $g(x) = \tan^{-1}\left(x^4\right)$

Expand and Cancel

Consider $$\sum_{n=0}^{\infty} x^{2n} + \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

Same idea as the
Telescoping Test

Consider $$\frac{1-\sum_{n=0}^{\infty} x^{2n}}{x}$$

What you cannot do

$$\left(\sum_{n=0}^{\infty} c_n x^n\right)^2 = (a_1 + a_2 + a_3 + \cdots)\,(a_1 + a_2 + a_3 + \cdots)$$

We cannot square a series.

Doing so would require infinite foiling or methods beyond the scope of this class.