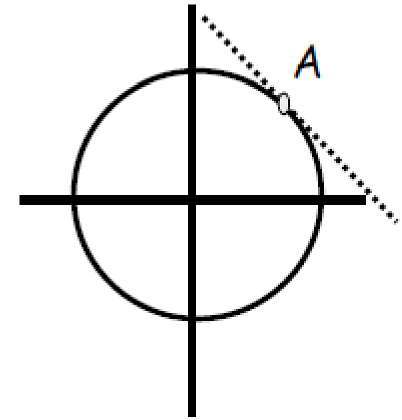
Derivatives of Polar Functions

Given
r = 2

$$\frac{dr}{d\theta} = 0$$



So, what does this mean?

$\frac{dr}{d\theta}$ Does not find the slope of the curve.

$\frac{dy}{dx}$ = Slope of the tangent

It finds the rate of change in *r* with respect to theta.

And in our circle, r did not change, which explains the 0.

If $\frac{dr}{d\theta}$ and r are opposite signs, then the particle is moving towards the pole at that angle.

Finding dy/dx

$$y = r\sin\theta \qquad x = r\cos\theta$$

Find $\dfrac{dy}{d\theta}$ and $\dfrac{dx}{d\theta}$

Hint: Both r and θ are variables. Use the product rule.

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta \qquad \frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

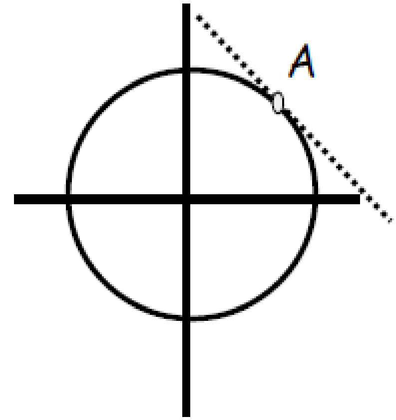
$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Written in function notation

$$\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$r = f(\theta)$$

So, let's go back to the original example…r = 2



As we said earlier, $\frac{dr}{d\theta}=0$

Now, let's find dy/dx:

$$\frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} = \frac{0\sin\theta + 2\cos\theta}{0\cos\theta - 2\sin\theta} = -\frac{\cos\theta}{\sin\theta} = -\cot\theta$$

At θ = π/4, dy/dx = -1

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$x = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2} \qquad y = 2\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}$$

Rectangular Equation of tangent line at θ = π/4: $y-\sqrt{2} = -\left(x-\sqrt{2}\right)$

Write a rectangular equation of the tangent line at $\theta = \frac{\pi}{2}$ for $r = 3(1 - \cos\theta)$ and describe the movement of a particle moving along $r(\theta)\ at\ \theta = \frac{\pi}{2}$.

For $r = 1 + \sin\theta$

a) Sketch the graph

b) Find dy/dx

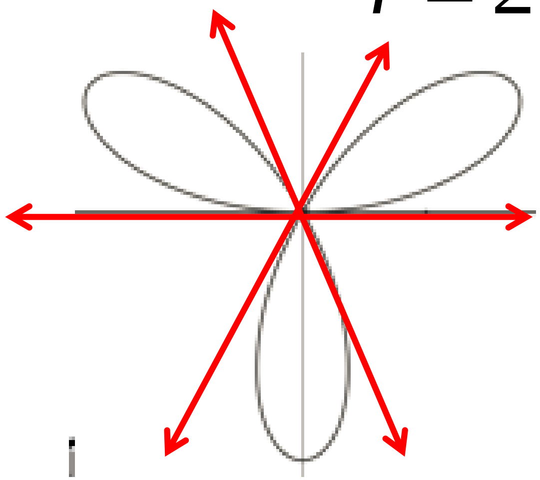
c) Find horizontal and vertical tangent lines.

Practice: p 689
7-15 odd write the equation of the tangent line
17, 19 Vertical and Horizontal tangents

Quiz tomorrow on what we've covered so far

Find the tangents lines to the curve at the pole.

$$r = 2\sin(3\theta), 0 \le \theta \le \pi$$



It looks like we have 3 tangents.

What does *r* equal at the pole?

r = 0

$$0 = 2\sin(3\theta)$$

$$0 = \sin(3\theta)$$

$$3\theta = 0, \pi, 2\pi$$

$$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

The point of tangency in Cartesian is (0, 0)

In polar, they are (0, 0), (0, π/3) and (0, 2π/3)

Before we go too far…

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

If r = 0

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + \cancel{r\cos\theta}}{\frac{dr}{d\theta}\cos\theta - \cancel{r\sin\theta}}$$

As long as r’ does not equal zero, it can be canceled out.

$$\frac{dy}{dx} = \frac{\cancel{\frac{dr}{d\theta}}\sin\theta}{\cancel{\frac{dr}{d\theta}}\cos\theta}$$

So, at the pole

$$\frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Find the tangents lines to the curve at the pole.

$$r = 2\sin(3\theta), 0 \le \theta \le \pi$$

$$\theta = 0, \frac{\pi}{3}, \frac{2\pi}{3}$$

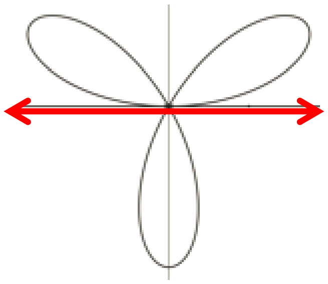
Find the slope of the line through each point

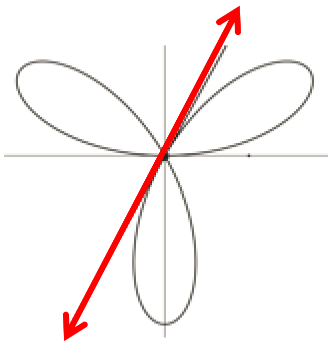
$$\frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

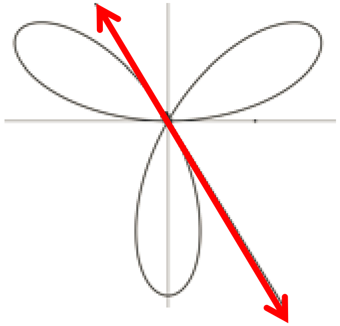
$$\frac{dy}{dx} = \tan 0 = 0$$

$$\frac{dy}{dx} = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\frac{dy}{dx} = \tan\frac{2\pi}{3} = -\sqrt{3}$$







Find the tangents lines to the curve at the pole.

$$r = 2\sin(3\theta), \quad 0 \le \theta \le \pi$$

$$\frac{dy}{dx} = \tan 0 = 0$$

$$\frac{dy}{dx} = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\frac{dy}{dx} = \tan\frac{2\pi}{3} = -\sqrt{3}$$

Cartesian lines through the pole, (0, 0) are easy now that we have slope.

$$y = 0$$
$$y = \sqrt{3}x$$
$$y = -\sqrt{3}x$$

Equations of line through the pole are even easier in polar.

$$\theta = 0$$
$$\theta = \frac{\pi}{3}$$
$$\theta = \frac{2\pi}{3}$$

Four Main Problem Types

- Finding dy/dx at specific point
- Finding equation of tangent at a point
- Finding equations of tangents at the pole
- Finding horizontal and vertical tangents

Practice:

- p689 #23, 27, 31, 35, 39, 43 Find tangents at the pole