

DERIVATIVES OF TRIG FUNCTIONS

Sections 2.2 and 2.3

$$\frac{d}{dx}(\sin x) = \lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h}$$

Use trig identity: $\sin(x + h) = \sin x \cos h + \cos x \sin h$

$$\frac{d}{dx}(\sin x) = \lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$

Rearrange 2nd and 3rd terms

$$= \lim_{h \rightarrow 0} \frac{\sin x \cos h - \sin x + \cos x \sin h}{h}$$

$$\boxed{\text{You prove } \frac{d}{dx}(\cos x) = -\sin x}$$

Factor out $\sin x$

$$= \lim_{h \rightarrow 0} \frac{\sin x (\cos h - 1) + \cos x \sin h}{h}$$

Split up terms

$$= \lim_{h \rightarrow 0} \frac{\sin x (\cos h - 1)}{h} + \frac{\cos x \sin h}{h}$$

Split up factors

$$= \lim_{h \rightarrow 0} \sin x \cdot \frac{\cos h - 1}{h} + \cos x \cdot \frac{\sin h}{h}$$

Evaluate known limits

$$= \lim_{h \rightarrow 0} \sin x \cdot 0 + \cos x \cdot 1 = \cos x$$

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$$

Use quotient rule

$$= \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x}$$

Simplify

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

Trig identity!

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\boxed{\text{Prove } \frac{d}{dx}(\cot x) = -\csc^2 x}$$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

Use quotient rule to prove

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

Examples

$$1. \quad y = x^2 \sin x$$

$$2. \quad y = \frac{\sec x}{x}$$

$$3. \quad y = \frac{\cos x}{1 - \sin x}$$