Euler's Method

Leonhard Euler made a huge number of contributions to mathematics, almost half after he was totally blind.

(When this portrait was made he had already lost most of the sight in his right eye.)

Leonhard Euler 1707 - 1783

It was Euler who originated the following notations:

$f(x)$ (function notation)

e (base of natural log)

π (pi)

i $\left(\sqrt{-1}\right)$

$\sum$ (summation)

Δy (finite change)

Leonhard Euler 1707 - 1783

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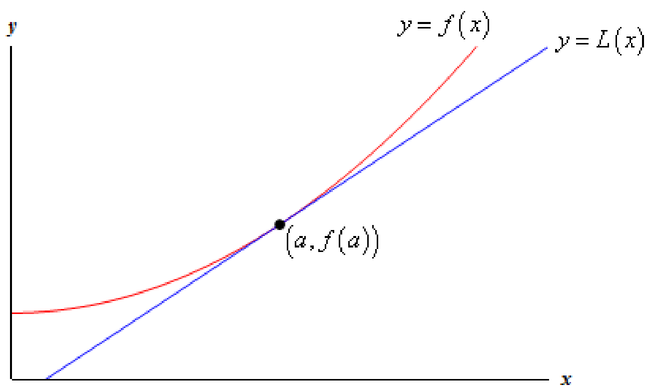
We found solution curves graphically using slope fields. Now we will estimate solutions numerically using Euler's Method.

<u>Remember</u>: A tangent line is a good approximation of a curve at a given point.



$$y - y_1 = m(x - x_1)$$

$$y = y_1 + m(x - x_1)$$

$$y_{new} = y_{old} + \frac{dy}{dx}\Delta x$$

Euler's method is an iterative process that steps closer and closer to the desired value. The smaller the steps, the better the approximation.

Example: $\frac{dy}{dx} = 2x$

$$y_{new} = y_{old} + \frac{dy}{dx}\Delta x$$

If the particular solution passes through (0, 1), estimate y(2).

Let's use a step size of 0.5.

$x_0 = 0$ $\quad y_0 = 1$ Find dy/dx. $\frac{dy}{dx} = 2(0) = 0$ Find y_1.

$x_1 = 0.5$ $\quad y_1 = 1 + 0(.5) = 1$ Repeat!

$x_2 = 1$

$x_3 = 1.5$

$x_4 = 2$

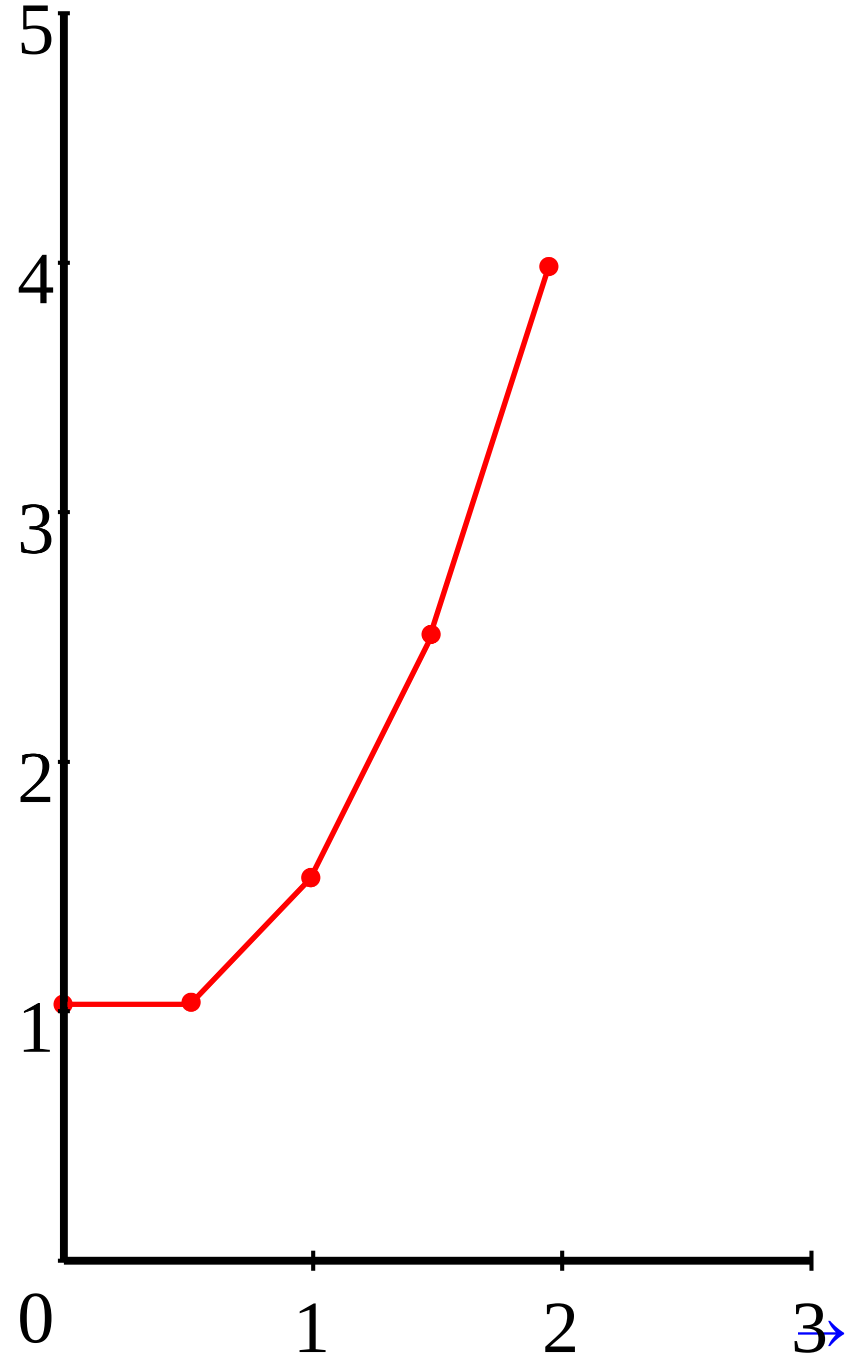
$$\frac{dy}{dx} = 2x \qquad f(0) = 1 \qquad dx = 0.5$$

$$f(0.5) \approx 1$$

$$f(1) \approx 1.5$$

$$f(1.5) \approx 2.5$$

$$f(2) \approx 4$$



$$\frac{dy}{dx} = 2x \qquad f(0) = 1 \qquad dx = 0.5$$

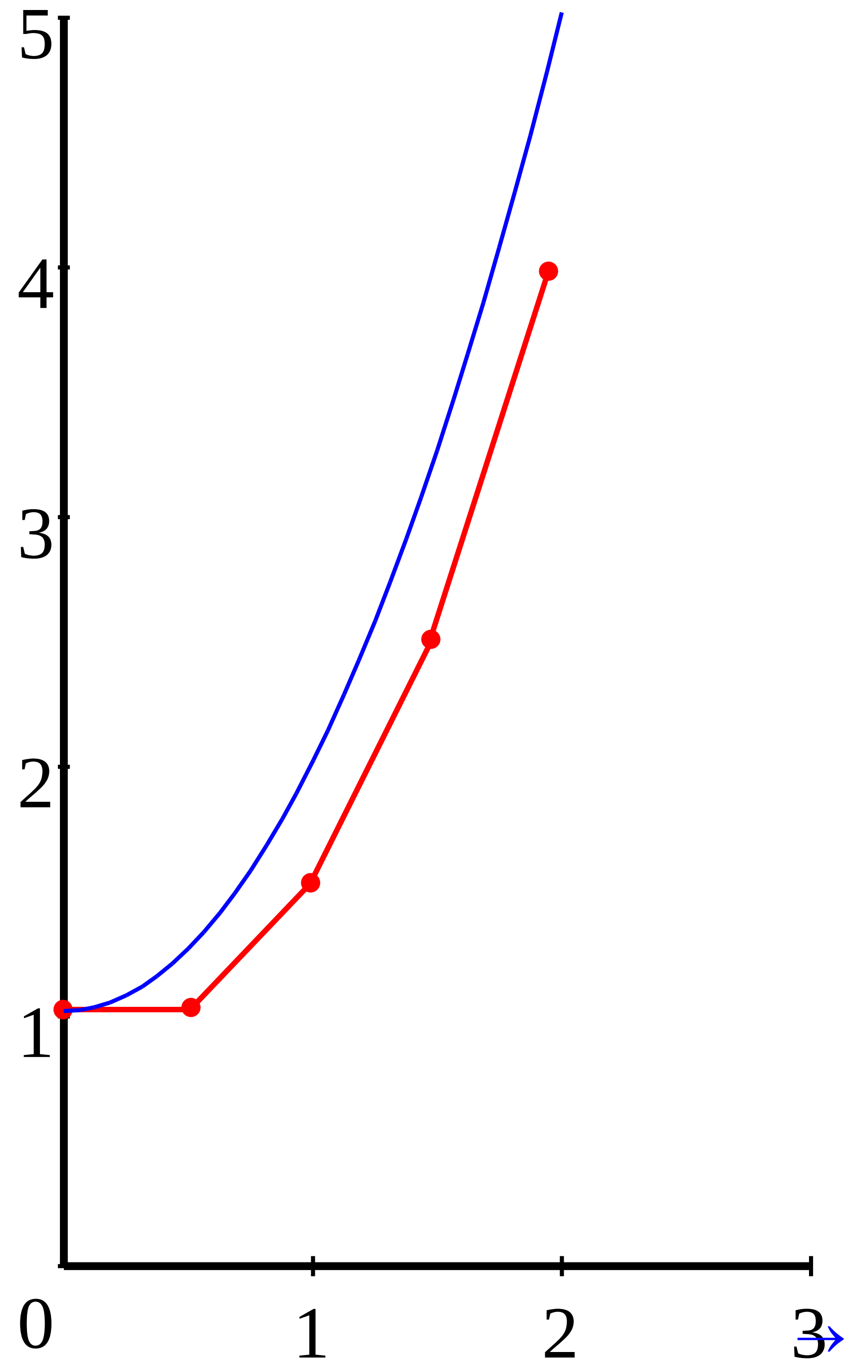
So how would this compare to the real solution?

$$y = x^2 + 1$$

How could we make the approximation better?

Answer: A smaller *dx* value

$$dx = 0.1$$



Example: $\frac{dy}{dx} = x - y$

$$y_{new} = y_{old} + \frac{dy}{dx}\Delta x$$

If the particular solution passes through (0, 1), estimate y(-0.4).

Let's use a step size of 0.2.

$$x_0 = 0 \qquad y_0 = 1 \qquad \frac{dy}{dx} = 0 - 1 = -1$$

$$x_1 = -0.2 \qquad y_1 = 1 + (-1)(-0.2) = 1.2 \qquad \frac{dy}{dx} = -0.2 - 1.2 = -1.4$$

$$x_2 = -0.4 \qquad y_2 = 1.2 + (-1.4)(-0.2) = 1.48$$

Since we are stepping backwards, $\Delta x = -0.2$

Try this one:

$$y' = x + y \qquad y(1) = 2 \qquad dx = 0.2$$

Approximate $y(2)$

Remember: $y_{new} = y_{old} + \frac{dy}{dx}\Delta x$

$$y' = x + y$$

x	y	y'
1	2	3
1.2	2.6	3.8
1.4	3.36	4.76
1.6	4.312	5.912
1.8	5.494	7.294
2	6.953	

Don’t round until the final answer!

Last one: $\frac{dy}{dx} = \frac{xy}{2} \quad y(0) = 3$

a. Appoximate y(0.4) using Euler's method with $\Delta x = 0.1$

b. Integrate to find the actual value of y(0.4)

Practice

- Copied p356 #15 and 16