Exam Review

Name _____

1 As a balloon in the shape of a sphere is being blown up, the radius is increasing $\frac{1}{\pi}$ inches per second. At what rate is the volume increasing when the radius is 1 inch?

(a) 4π in.^{3}/sec (b) 3 in.^{3}/sec (c) 4 in.^{3}/sec

(d) 3π in.^{3}/sec (e) None of these

2 A side of a square is increasing at the rate of 2 feet per minute. Find the rate at which the area is increasing when the side is 7 feet.

(a) 28 ft^2/min (b) 49 ft^2/min (c) 14 ft^2/min

(d) 28π ft^2/min (e) None of these

3 A particle moves on the curve $y = \frac{3}{x^2 + 4}$ such that $\frac{dy}{dt} = 6$. Find the instantaneous rate of change of x with respect to t when $x = 2$.

(a) -128 (b) -32 (c) 32 (d) 128 (e) None of these

4 Let $f(x)$ be a polynomial function such that $f(-2) = 5$, $f'(-2) = 0$, and $f''(-2) = 3$. The point $(-2, 5)$ is a _____ of the graph of f.

(a) Relative maximum (b) Relative minimum (c) Intercept

(d) Point of inflection (e) None of these

5 Let $f(x)$ be a polynomial function such that $f(4) = -1$, $f'(4) = 2$, $f''(4) = 0$. If $x < 4$, then $f''(x) < 0$ and if $x > 4$, then $f''(x) > 0$. The point $(4, -1)$ is a _____ of the graph of f.

(a) Relative maximum (b) Relative minimum (c) Critical number

(d) Point of inflection (e) None of these

6 Find all points of inflection of the graph of the function $f(x) = x^4 + x^3$.

(a) $(0, 0)$ and $\left(-\frac{1}{2}, -\frac{1}{16}\right)$ (b) $\left(-\frac{1}{2}, -\frac{1}{16}\right)$ (c) $(0, 0)$

(d) $(0, 0)$ and $\left(-\frac{3}{4}, -\frac{27}{256}\right)$ (e) None of these

7 Evaluate $\lim_{x \to \infty} \frac{x^2}{\ln x}$.

(a) 0 (b) 1 (c) ∞

(d) Does not exist (e) None of these

8 Evaluate $\lim_{x \to 1} \frac{\ln x}{x - 1}$.

9 Evaluate $\lim_{x \to \infty} \frac{\ln x}{x^3}$.

10 Evaluate $\lim_{x \to 0} \frac{8e^{x/8} - (8 + x)}{x^2}$.

(a) $\frac{1}{16}$ (b) ∞ (c) $\frac{1}{2}$

(d) 0 (e) None of these

11 Evaluate $\lim_{x \to 0} \frac{3e^{x/3} - (3 + x)}{x^2}$

(a) 0 (b) ∞ (c) $\frac{1}{6}$

(d) $\frac{1}{2}$ (e) None of these

12 Identify the definite integral that represents the arc length of the curve $y = \sqrt{x}$ over the interval $[0, 3]$.

(a) $\int_0^3 \sqrt{1 + \frac{1}{4x}}\,dx$ (b) $\int_0^3 \sqrt{1 + \frac{1}{2x}}\,dx$ (c) $\int_0^3 \sqrt{1 + x}\,dx$

(d) $\int_0^3 \sqrt{x}\,dx$ (e) None of these

13 Find the partial fraction decomposition: $\frac{9x^2 + x - 1}{x^2(x + 1)}$.

(a) $\frac{2}{x} - \frac{1}{x^2} + \frac{7}{x + 1}$ (b) $\frac{20}{x} - \frac{1}{x^2} - \frac{11}{x + 1}$ (c) $\frac{9}{x} + \frac{1}{x^2} - \frac{1}{x + 1}$

(d) $\frac{-1}{x^2} + \frac{9}{x + 1}$ (e) None of these

14 Evaluate: $\int \frac{3x+4}{(x^2+4)(3-x)}\,dx$.

(a) $\frac{1}{2}\ln(x^2+4) + \ln|3-x| + C$

(b) $\frac{1}{2}\arctan\frac{x}{2} + \ln|3-x| + C$

(c) $\frac{1}{2}\arctan\frac{x}{2} - \ln|3-x| + C$

(d) $\ln\left|\frac{\sqrt{x^2+4}}{3-x}\right| + C$

(e) None of these

15 Evaluate: $\int xe^{2x}\,dx$.

(a) $2e^{2x}(x-2) + C$

(b) $x^2e^{x^2} + C$

(c) $\frac{e^{2x}}{4}(2x-1) + C$

(d) $\frac{1}{2}x^2e^{2x} + C$

(e) None of these

16 Evaluate: $\int x\sin x\,dx$.

(a) $\cos x - x\sin x + C$

(b) $\sin x + x\cos x + C$

(c) $\cos x + x\sin x + C$

(d) $\sin x - x\cos x + C$

(e) None of these

17 Find the arc length of the curve given by $x = t^2$ and $y = 2t^2 + 1$, $1 < t < 3$.

(a) $16\sqrt{5}$

(b) 40

(c) 24

(d) $8\sqrt{5}$

(e) None of these

18 Find $\frac{d^2y}{dx^2}$ for the curve given by $x = 2\cos\theta$ and $y = \sin\theta$.

(a) $-\frac{1}{4}\csc^3\theta$

(b) $\frac{1}{2}\csc^2\theta$

(c) $-2\sec^2\theta$

(d) $\frac{1}{2}\cot\theta\csc\theta$

(e) None of these

19 Find $\frac{d^2y}{dx^2}$ for the curve given by $x = \frac{1}{2}t^2$ and $y = t^2 + t$.

(a) $-\frac{1}{t^2}$

(b) $-\frac{1}{t^3}$

(c) $\frac{2t+1}{t}$

(d) 2

(e) None of these

20 Find $\frac{dy}{dx}$ for the curve given by $x = t^2$ and $y = \sqrt{t-1}$.

(a) $\frac{1}{4t\sqrt{t-1}}$

(b) $\frac{1}{2\sqrt{t-1}}$

(c) $\frac{t}{\sqrt{t-1}}$

(d) $2t$

(e) None of these

21 Find $\frac{dy}{dx}$ for the curve given by $x = \sqrt{t}$ and $y = (t-1)^3$.

(a) $3(t-1)^2$

(b) $\frac{1}{6\sqrt{t}(t-1)^2}$

(c) $\frac{6(t-1)^2}{\sqrt{t}}$

(d) $6\sqrt{t}(t-1)^2$

(e) None of these

22 Find the equation of the tangent line for the curve given by $x = 2t$ and $y = t^2 + 5$ at the point where $t = 1$.

(a) $y = 2x + 2$

(b) $y = tx - 2t + 6$

(c) $y = x + 4$

(d) $y = x - 4$

(e) None of these

23 Find the equation of the tangent line for the curve given by $x = 3t - 1$ and $y = t^2$ at the point where $t = 1$.

(a) $2x - 3y - 1 = 0$

(b) $3y = 2x + 1$

(c) $y - 1 = \frac{2}{3}t(x-2)$

(d) $y = 2x - 3$

(e) None of these

24 Eliminate the parameter and find a corresponding rectangular equation: $x = 3t^2$ and $y = 2t + 1$.

(a) $2x^2 + 3y^2 - 1 = 0$

(b) $2x - 3y + 3 = 0$

(c) $3y^2 - 4x + 1 = 0$

(d) $3y^2 - 4x - 6y + 3 = 0$

(e) None of these

25 Eliminate the parameter and find a corresponding rectangular equation: $x = 2\cos\theta$ and $y = \cos^2\theta$.

(a) $x + y = \cos\theta(2 + \cos\theta)$

(b) $x - 2y = 0$

(c) $y = \left(1 - \frac{x}{2}\right)^2$

(d) $x^2 = 4y$

(e) None of these

26 Find the corresponding rectangular coordinates for the polar point $\left(-4, \frac{\pi}{6}\right)$.

(a) $(-2\sqrt{3}, -2)$

(b) $(-2, -2\sqrt{3})$

(c) $(2\sqrt{3}, 2)$

(d) $(2, 2\sqrt{3})$

(e) None of these

27 Calculate the area inside the cardioid $r = 1 + \cos\theta$.

(a) 3π (b) $\frac{3\pi}{4}$ (c) $\frac{3\pi}{2}$

(d) $\frac{\pi}{2}$ (e) None of these

28 Calculate the area of common interior of $r = 2\sin\theta$ and $r = 2\cos\theta$.

(a) π (b) $\frac{\pi}{2} - 1$ (c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2} - 2$ (e) None of these

29 Calculate the area enclosed by the graph of $r = 3\cos 3\theta$.

(a) π (b) $\frac{9\pi}{8}$ (c) $\frac{9\pi}{4}$

(d) $\frac{9\pi}{2}$ (e) None of these

30 Find the slope of the tangent line for the curve $r = 2\cos 3\theta$ at the point where $\theta = \frac{\pi}{6}$.

(a) -6 (b) $\frac{1}{\sqrt{3}}$ (c) $-\sqrt{3}$

(d) 1 (e) None of these

31 Identify the special polar graph given by the equation: $r = 2 - 3\sin\theta$.

(a) Limaçon with inner loop (b) Cardioid (c) Dimpled limaçon

(d) Rose curve (e) None of these

32 Investigate the convergence or divergence of the series using the Ratio Test: $\sum_{n=1}^{\infty} \frac{n^7}{7^n}$.

(a) Converges by the Ratio Test (b) Diverges by the Ratio Test

(c) The Ratio Test does not apply. (d) The Ratio Test is inconclusive.

(e) None of these

33 Choose the test which could be used to prove the divergence of the series $\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$.

(a) Geometric Series Test (b) p-Series Test (c) Ratio Test

(d) nth-Term Test for Divergence (e) None of these

34 Investigate the convergence or divergence of the series using the Ratio Test: $\sum_{n=1}^{\infty} \frac{n!}{(n+5)!}$.

(a) The Ratio Test does not apply. (b) Converges by the Ratio Test

(c) Diverges by the Ratio Test (d) The Ratio Test is inconclusive.

(e) None of these

35 Choose the test which could be used to show that the series $\sum_{n=1}^{\infty} \left(\frac{2n-1}{3n+5}\right)^n$ converges.

(a) Root Test (b) Ratio Test (c) Geometric Series Test

(d) p-Series Test (e) None of these

36 Find the Taylor polynomial of degree two for the function $f(x) = \ln(x^2 + 4)$ centered at 0.

(a) $\ln 4 + \frac{5}{32}x^2$ (b) $\ln 4 + \frac{x}{2} + \frac{x^2}{4}$ (c) $\ln 4 + \frac{x^2}{2}$

(d) $\ln 4 + \frac{x^2}{4}$ (e) None of these

37 Find the third degree Taylor polynomial centered at $c = 1$ for the function $f(x) = e^{3x}$.

(a) $e^3 + 3e^3x + \frac{9e^3x^2}{2} + \frac{9e^3x^3}{2}$

(b) $e^3x + \frac{3e^3x^2}{2} + \frac{3e^3x^3}{2} + \frac{9e^3x^4}{8}$

(c) $e^3(x+1) + \frac{3e^3(x+1)^2}{2} + \frac{3e^3(x+1)^3}{2} + \frac{9e^3(x+1)^4}{8}$

(d) $e^3 + 3e^3(x-1) + \frac{9e^3(x-1)^2}{2} + \frac{9e^3(x-1)^3}{2}$

38 Choose the series that diverges.

(a) $\sum_{n=0}^{\infty} \frac{n!}{3n! - 1}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^6}$ (c) $\sum_{n=0}^{\infty} 5\left(\frac{1}{10}\right)^n$

(d) $\sum_{n=0}^{\infty} \frac{n}{2^n}$ (e) None of these

39 Choose the series that converges.

(a) $\sum_{n=1}^{\infty} [2 + (-1)^n]$ (b) $\sum_{n=1}^{\infty} \frac{2^n}{(n+1)!}$ (c) $\sum_{n=0}^{\infty} 5\left(\frac{3}{2}\right)^n$

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ (e) None of these

40 Find the fourth degree Taylor polynomial centered at $c = 3$ for the function $f(x) = \ln(x - 2)$.

(a) $1 + x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$ (b) $(x-3) - \frac{(x+3)^2}{2!} + \frac{(x+3)^3}{3!} - \frac{(x+3)^4}{4!}$

(c) $(x-3) - \frac{(x-3)^2}{2} + \frac{(x-3)^3}{3} + \frac{(x-3)^4}{4}$ (d) $(x-3) - \frac{(x-3)^2}{2!} + \frac{(x-3)^3}{3!} - \frac{(x-3)^4}{4!}$

(e) None of these

41 Investigate the series $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^n}$ for convergence or divergence.

(a) Converges by Ratio Test (b) Diverges by nth-Term Test for Divergence

(c) Converges by Root Test (d) Diverges by Integral Test

(e) None of these

42 Investigate the convergence or divergence of the series using the Ratio Test: $\sum_{n=1}^{\infty} \frac{n!}{3^n}$.

(a) The Ratio Test is inconclusive. (b) Diverges by the Ratio Test

(c) Converges by the Ratio Test (d) The Ratio Test does not apply.

(e) None of these