

# Unit 4

# More with Derivatives

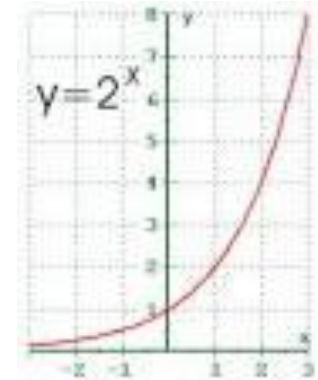
Sections 7.2, 7.3, 7.4

Exponential and Logarithmic Functions



# Graph $y=2^x$ on your calculator

- ▶ Notice the tangent lines get steeper and steeper
- ▶ Now graph the slopes of the tangent lines using your calculator

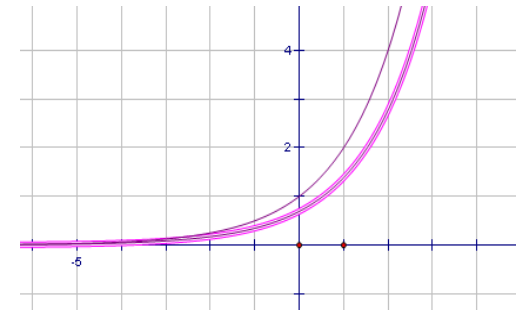


- ▶ Y=, Y2=Math, 8:nderiv( , nderiv( $2^x$ , x, x)

Asking the calculator to graph the derivative of  $2^x$ , with respect to  $x$ , for all  $x$ 's.

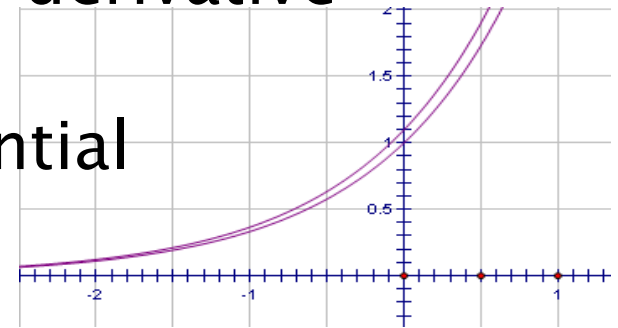
$$\left. \frac{d}{dx} (2^x) \right|_{x=x}$$

$Y'$  is an exponential function  
below  $y=2^x$



▶ Repeat by graphing  $y = 3^x$  and its derivative

▶ Derivative of  $y = 3^x$  is an exponential  
function above  $y = 3^x$



Repeat with different bases until you reach an  
exponential function that is its own  
derivative.

$$y = 2.5^x \quad y = 2.75^x \quad y = 2.7^x$$

$$y = 2.718281828\dots^x = e^x \text{ !!!!!}$$

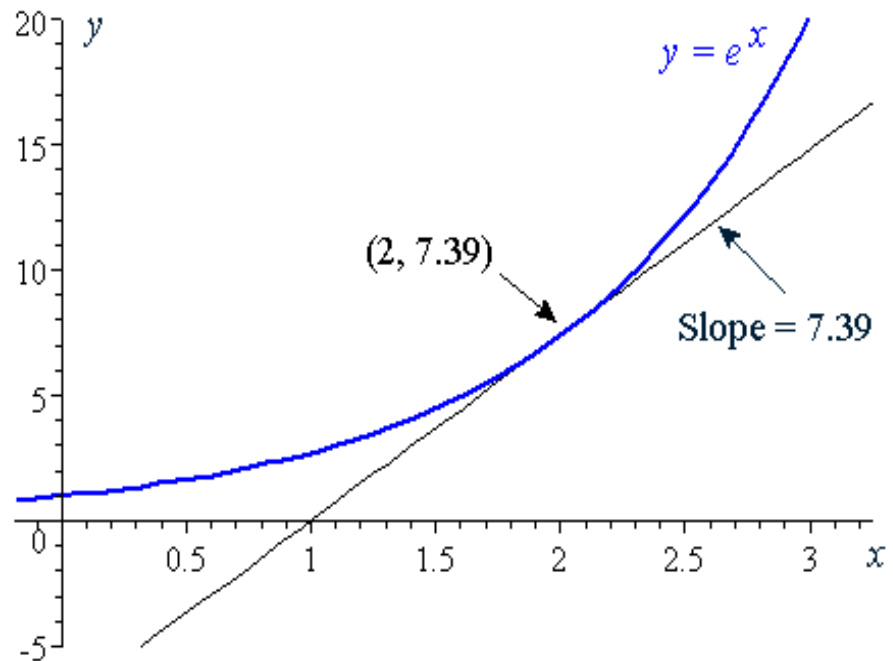
- ▶  $Y = e^x$  grows at a rate equal to its value.

$$\frac{d}{dx}(e^x) = e^x$$

- ▶ Examples:

$$y = e^{3x}$$

$$y = e^{x^2}$$



# Find the derivative of $y = a^x$

▶  $y = a^x$

▶  $y = e^{\ln a^x}$

▶  $y = e^{x \ln a}$

▶  $y' = e^{x \ln a} \cdot \ln a$

▶  $y' = e^{\ln a^x} \cdot \ln a$

▶  $y' = a^x \ln a$

$\frac{d}{dx} (a^x) = a^x \ln a$
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# Examples: Find the derivatives.

▶  $y = 5^x$

$y = x^5$

$y = 6^{3x}$

# Find the derivative of $y = \ln x$

▶  $y = \ln x$

▶  $e^y = e^{\ln x}$

▶  $e^y = x$

▶  $\frac{d}{dx} (e^y = x)$

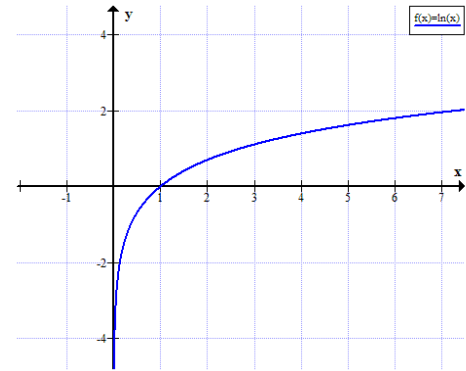
▶  $e^y \frac{dy}{dx} = 1$

▶  $\frac{dy}{dx} = \frac{1}{e^y}$

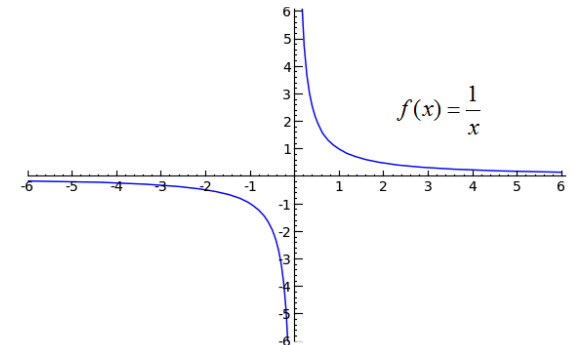
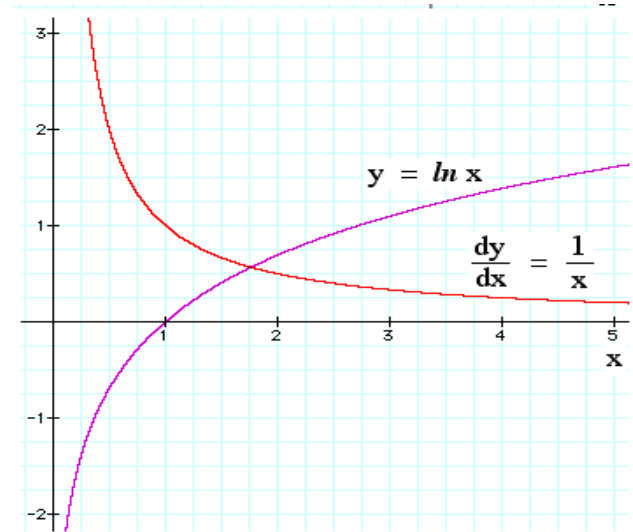
$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$\frac{d}{dx} (\ln x) = \frac{1}{x}$
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# Graph $f(x) = \ln x$



- ▶ What is its domain?
- ▶ D:  $(0, \infty)$
- ▶ Graph derivative  $f'(x) = 1/x$  using “nder”
- ▶ Domain of a derivative cannot be larger than the original function's domain.
- ▶ Notice negative values are not shown.
- ▶ Domain of  $f'(x)$  is D:  $(0, \infty)$
- ▶ Domain of  $g(x) = 1/x$  is D:  $(-\infty, 0)(0, \infty)$





Find the derivative of  $y = \log_a x$

▶  $y = \log_a x$

▶  $y = \frac{\ln x}{\ln a}$

▶  $y = \frac{1}{\ln a} \cdot \ln x$

▶  $y' = \frac{1}{\ln a} \cdot \frac{1}{x}$

$\frac{d}{dx} (\log_a x) = \frac{1}{x \ln a}$
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# Examples:

$$y = \ln(\sin x)$$

$$y = \sqrt{\ln x}$$

# Examples:

$$\frac{d}{dx} \left[ \ln \left( \frac{x+1}{\sqrt{x-2}} \right) \right]$$