Factorials !

$n!$ = product of all integers from 1 to n

$0! = 1$

$1! = 1$

Examples

- $4! =$
- $6! =$
- $n! =$
- $(n - 2)! =$
- $(n + 1)! =$

Simplifying Factorials

- $\dfrac{5!}{7!}$
- $\dfrac{8!}{11!}$
- $\dfrac{59!}{57!}$

Simplifying Factorials

- $$\frac{(n-2)!}{n!}$$

- $$\frac{(2n-1)!}{(2n+1)!}$$

Factorial Practice

- Answers

10) $n(n-1)$

11) $\frac{1}{n+1}$

12) $n(n-1)(n-2)$

13) $\frac{1}{2n+1}$

14) $\frac{1}{(2n+3)(2n+2)}$

15) $(2n+2)(2n+1)$

19) $10!$

20) $3(15!)$

21) $5(40!)$

Section 9.1 Sequences

A <u>sequence</u> is a list or set whose domain is a set of positive integers

$\{a_1, a_2, \dots\}$ where $n \epsilon Z^+$

Find the first three terms of each sequence

$$a_n = \frac{n}{n^2 + 1}$$

$$a_n = \frac{n^2}{2^n - 1}$$

$$a_n = \frac{2^n}{n!}$$

$$a_n = 3 + (-1)^n$$

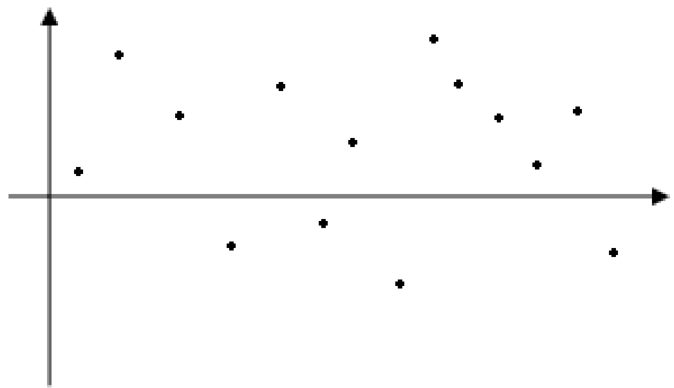
- A sequence is increasing if $a_{n+1} > a_n$ $\forall$ (for all) $n \geq 1$

Convergence vs. Divergence

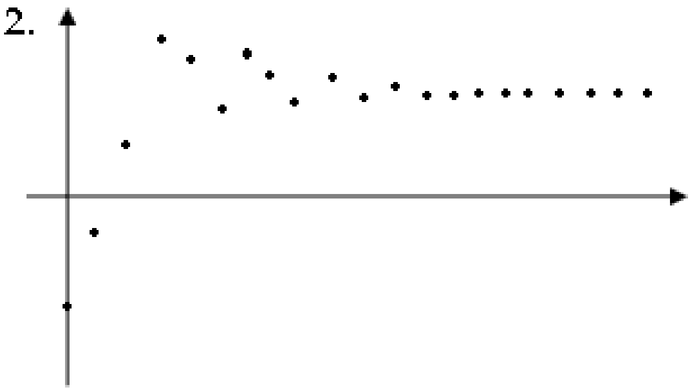
- If $\lim_{n\to\infty} a_n = L$, the sequence converges to L.
- Otherwise the sequence diverges.
- A divergence sequence may approach $\pm\infty$ or may be oscillating between two numbers.

Limit of a sequence

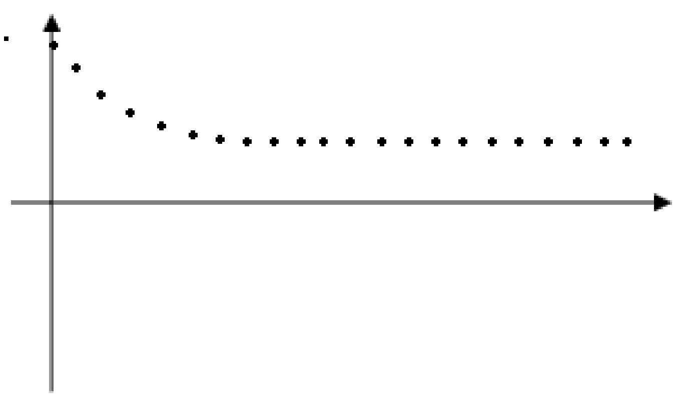
1.



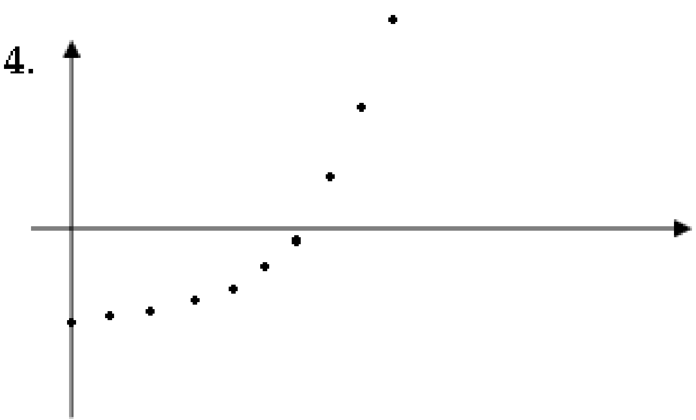
2.



3.



4.



Examples

1. $\{2, 4, 6, \ldots\}$ Diverges because each element is larger than the one before and the limit increases without bound

2. $\left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots\right\}$ Converges to the number 0

3. $a_n = \sin n$ Diverges. The sequence is bounded between 1 and -1. It oscillates and never settles on a single number.

4. $a_n = \frac{(-1)^{n+1}}{n} = \left\{1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \cdots\right\}$ Converges to 0 even though it oscillates between positive and negative values.

Determine if each sequence converges or diverges. If it converges, find the limit.

1. $a_n = \frac{1}{n}$ $\quad \lim_{n\to\infty} \frac{1}{n} = 0$ Converges to 0

2. $a_n = 1 + \frac{(-1)^n}{n}$ Converges to 1

3. $a_n = \frac{2n^2}{3n^3-1}$ Converges to 0

4. $a_n = \frac{3n^4+5}{4n^4-7n^2+9}$ Converges to 3/4

5. $a_n = \frac{n^2-1}{n}$ Diverges

6. $a_n = \cos(2n)$ Diverges

7. $a_n = (-1)^{2n}$ $\{1, 1, \ldots\}$ Converges to 1

8. $a_n = \frac{\ln n}{n}$ Converges to 0

Classwork/Homework

p. 552 #2, 4, 6, 9, 15,
25 – 37 odd, 47, 51, 55