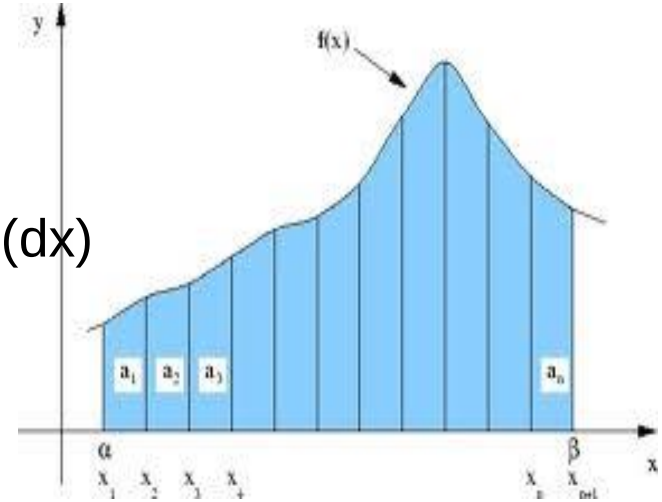
Finding Area inside Polar Regions

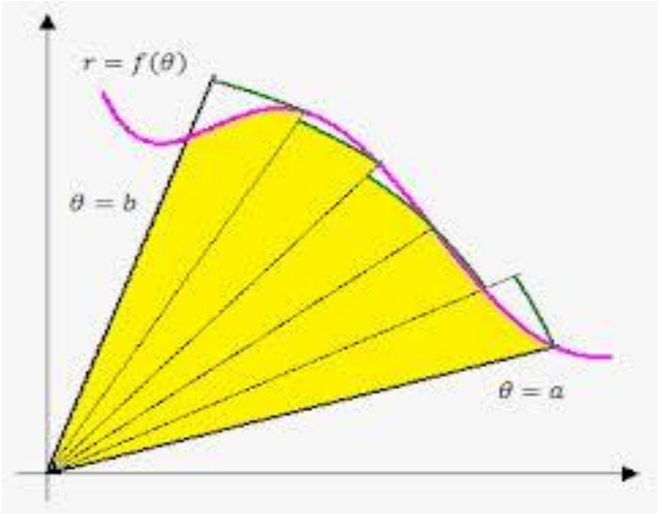
Remember to find area under a curve you add up an infinite number of rectangles



Area of a rectangle = length (f(x)) * width (dx)

$$\int_a^b f(x)\,dx$$

To find area enclosed by a polar curve, you add up an infinite number of sectors.



Area of a sector = fraction of a circle

$$= \frac{\theta}{2\pi} * \pi r^2$$

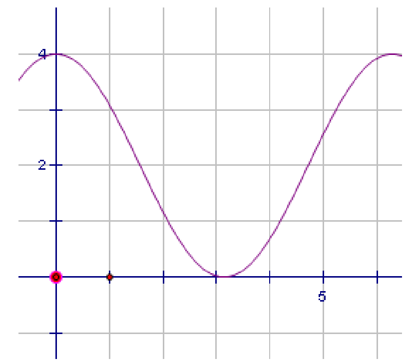
$$= \frac{1}{2} r^2 \theta$$

Area enclosed by a polar region $= \int_a^b \frac{1}{2} r^2 \, d\theta$

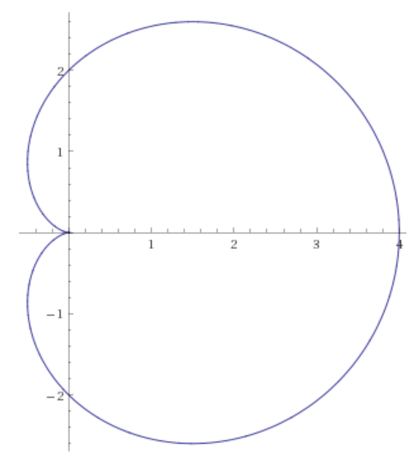
Example 1:

Find the area enclosed by $r = 2 + 2\cos\theta$

First sketch the graph to find what interval makes a full curve either by hand or on the calculator.



$$\frac{1}{2}\int_0^{2\pi}\left(2+2\cos\theta\right)^2 d\theta$$ =18.849 = 6π

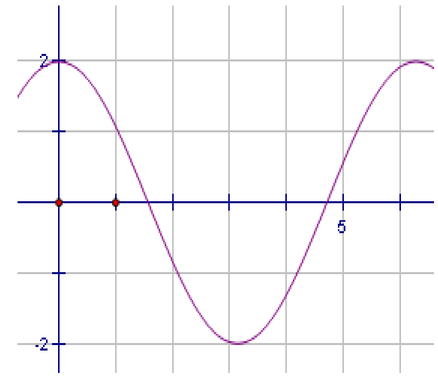


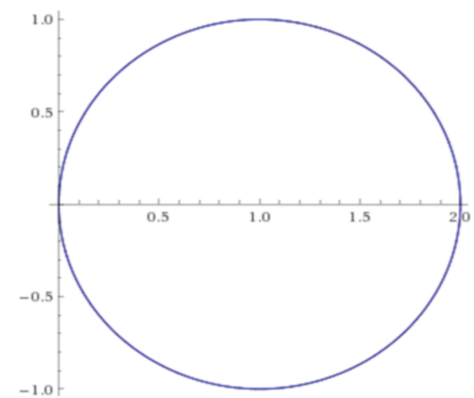
Example 2:

Find the area within $r = 2\cos\theta$

$$Area = \frac{1}{2}\int_{0}^{\pi}\left(2\cos\theta\right)^{2} d\theta$$

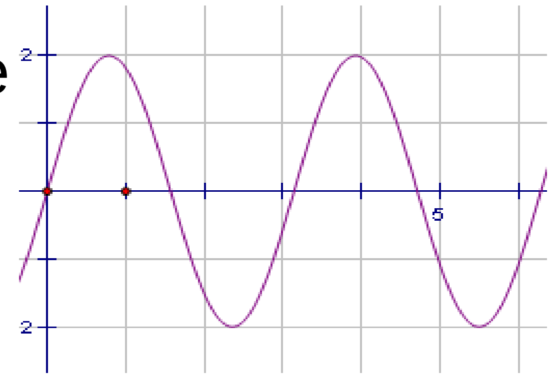
$$Area = 3.1415 = \pi$$





Make a full circle in π

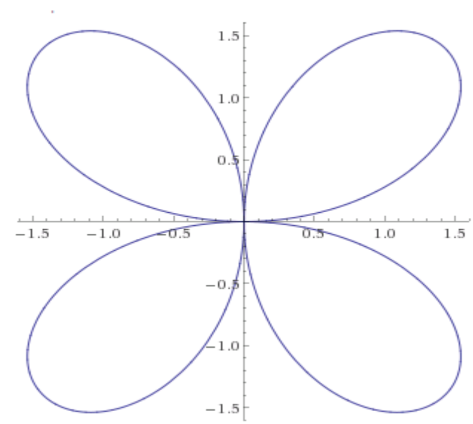
Example 3:
Find the area inside $r = 2\sin(2\theta)$



Use symmetry; find the area of one petal, them multiply by 4.

One petal is drawn from 0 to π/2

$$Area = 4 \bullet \frac{1}{2}\int_0^{\pi/2} \left[2\sin(2\theta)\right]^2 d\theta$$

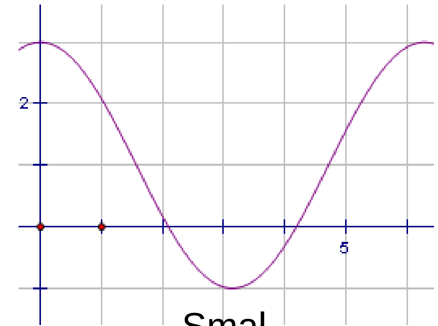


Area = 6.283 or 2π

Example 4: Find the area inside the small loop of $r = 1 + 2\cos\theta$

Be careful!

$$\int_0^{2\pi} 1 + 2\cos\theta\, d\theta$$ = inner + outer loop



Small loop

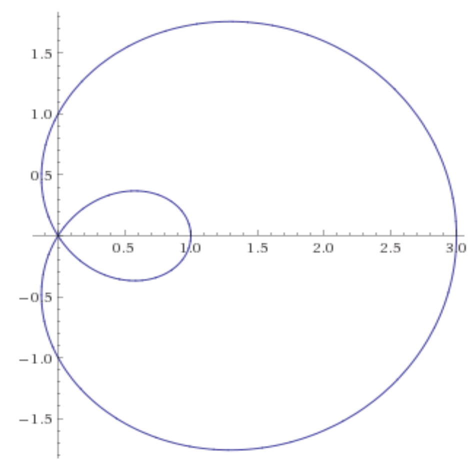
The inner loop lies on top of outer loop.

$$1 + 2\cos\theta = 0$$

$$\cos\theta = \frac{-1}{2}$$

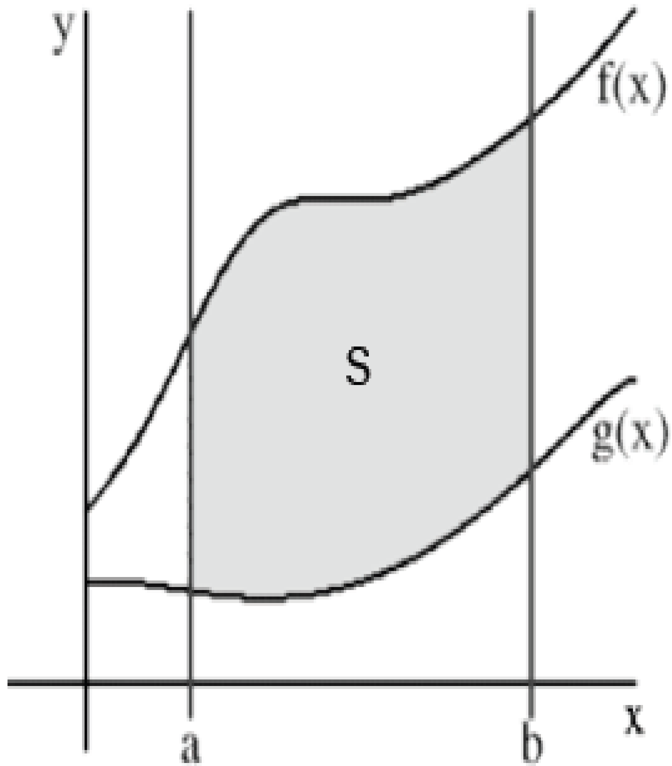
$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\frac{1}{2}\int_{2\pi/3}^{4\pi/3} (1 + 2\cos\theta)^2\, d\theta$$



Area = 0.544

Remember to find the area between curves on a rectangular grid:

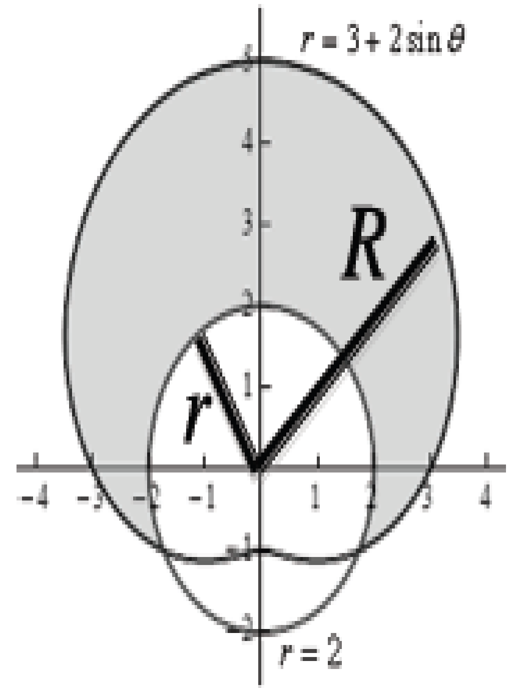


$$S = \int_a^b top - bottom$$

$$S = \int_a^b f(x) - g(x)\, dx$$

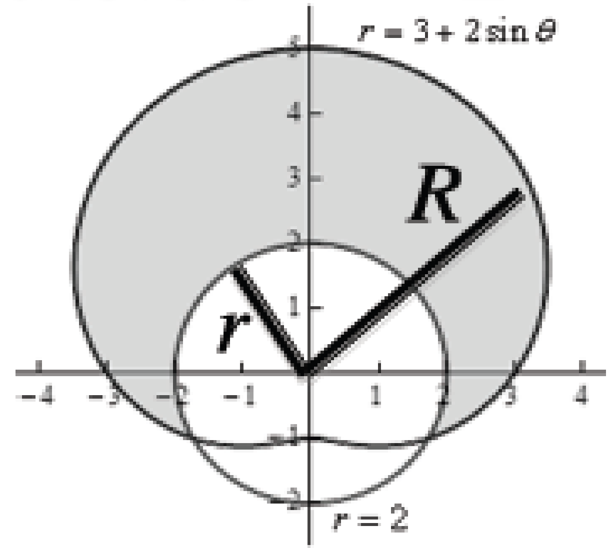
To find the area between polar curves:

$$A = \frac{1}{2}\int_{\alpha}^{\beta} R^2 - r^2 \, d\theta$$



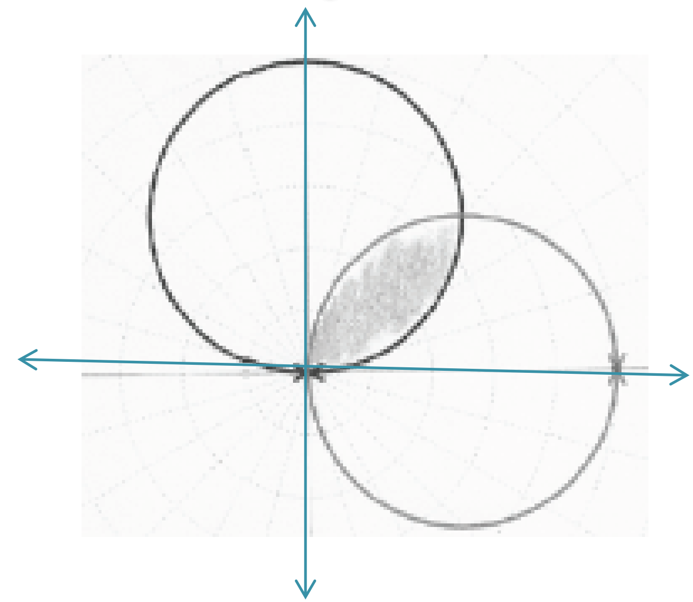
Example 5: Find the area inside $r = 3 + 2\sin\theta$ but outside r = 2

Just like finding areas between Cartesian curves, the limits of integration are the intersection of the curves.



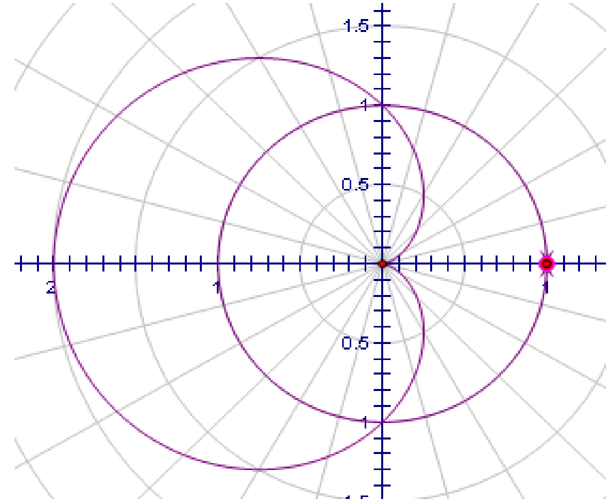
Example 6:

Find the area enclosed by r = sin ϴ and r = cos ϴ .

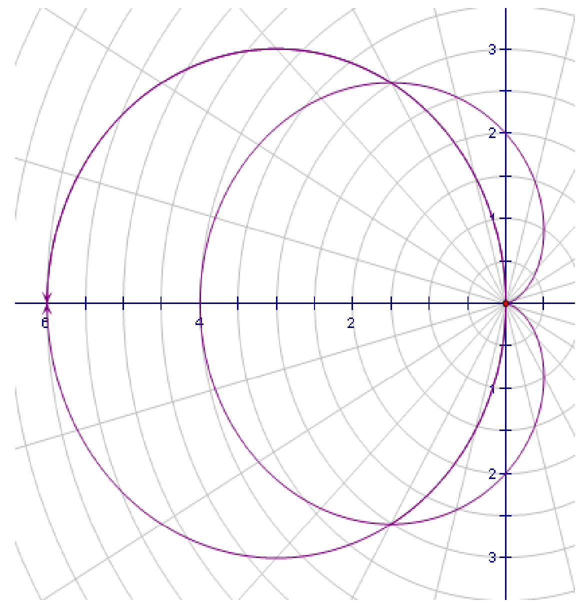


Try:

1. Find the area of the region inside r = 1 and outside
   $r = 1 - \cos \theta$.



2. Find the area of the region inside $r = -6 \cos \theta$ and
   $r = 2 - 2 \cos \theta$.



Homework:
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