## Quick Review 8.3

In Exercises 1–4, evaluate the integral.

1. $\int_0^3 \frac{dx}{x+3}$
2. $\int_{-1}^1 \frac{x\,dx}{x^2+1}$
3. $\int \frac{dx}{x^2+4}$
4. $\int \frac{dx}{x^4}$

In Exercises 5 and 6, find the domain of the function.

5. $g(x) = \frac{1}{\sqrt{9-x^2}}$
6. $h(x) = \frac{1}{\sqrt{x-1}}$

## Section 8.3 Exercises

In Exercises 1–6, **(a)** state why the integral is improper or involves improper integrals. Then, **(b)** determine whether the integral converges or diverges, and **(c)** evaluate the integral if it converges.

1. $\int_0^\infty \frac{dx}{x^2+1}$
2. $\int_0^1 \frac{dx}{\sqrt{x}}$
3. $\int_{-8}^1 \frac{dx}{x^{1/3}}$
4. $\int_{-\infty}^\infty \frac{2x\,dx}{(x^2+1)^2}$
5. $\int_0^{\ln 2} x^{-2}e^{1/x}\,dx$
6. $\int_0^{\pi/2} \cot\theta\,d\theta$

In Exercises 7–26, evaluate the integral or state that it diverges.

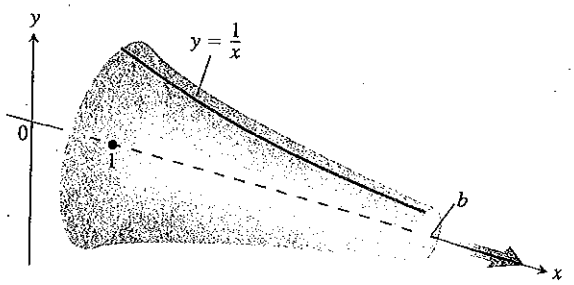
7. $\int_1^\infty \frac{dx}{x^{1.001}}$
8. $\int_{-1}^1 \frac{dx}{x^{2/3}}$
9. $\int_0^4 \frac{dr}{\sqrt{4-r}}$
10. $\int_0^1 \frac{dr}{r^{0.999}}$
11. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
12. $\int_{-\infty}^2 \frac{2\,dx}{x^2+4}$
13. $\int_{-\infty}^{-2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right]dx$
14. $\int_2^\infty \left[\frac{3}{t-1} - \frac{3}{t}\right]dt$
15. $\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta$
16. $\int_0^2 \frac{s+1}{\sqrt{4-s^2}}\,ds$
17. $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$
18. $\int_1^\infty \frac{dx}{x\sqrt{x^2-1}}$
19. $\int_1^2 \frac{ds}{s\sqrt{s^2-1}}$
20. $\int_{-1}^\infty \left[\frac{1}{\theta+2} - \frac{1}{\theta+3}\right]d\theta$
21. $\int_0^\infty \frac{16\tan^{-1}x}{1+x^2}\,dx$
22. $\int_{-1}^4 \frac{dx}{\sqrt{|x|}}$
23. $\int_{-\infty}^0 \theta e^\theta\,d\theta$
24. $\int_0^\infty 2e^{-\theta}\sin\theta\,d\theta$
25. $\int_{-\infty}^\infty e^{-|x|}\,dx$
26. $\int_0^1 x\ln x\,dx$

In Exercises 27–46, use integration, the direct comparison test, or the limit comparison test to determine whether the integral converges or diverges.

27. $\int_0^{\pi/2} \tan\theta\,d\theta$
28. $\int_0^\pi \frac{\sin\theta\,d\theta}{\sqrt{\pi-\theta}}$
29. $\int_{-\infty}^\infty 2x\,e^{-x^2}\,dx$
30. $\int_0^4 \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$
31. $\int_0^\pi \frac{dt}{\sqrt{t}+\sin t}$
32. $\int_4^\infty \frac{dx}{\sqrt{x}-1}$
33. $\int_1^\infty \frac{dx}{x^3+1}$
34. $\int_0^2 \left[\frac{1}{2x+2} - \frac{1}{2x-2}\right]dx$
35. $\int_0^2 \frac{dx}{1-x}$
36. $\int_{-1}^1 \ln|x|\,dx$
37. $\int_1^\infty \frac{d\theta}{1+e^\theta}$
38. $\int_2^\infty \frac{dx}{\sqrt{x^2-1}}$
39. $\int_1^\infty \frac{\sqrt{x+1}}{x^2}\,dx$
40. $\int_0^\infty \frac{dx}{\sqrt{x}}$
41. $\int_\pi^\infty \frac{2+\cos x}{x}\,dx$
42. $\int_\pi^\infty \frac{1+\sin x}{x^2}\,dx$
43. $\int_{-\infty}^\infty \frac{dx}{e^x+e^{-x}}$
44. $\int_{-\infty}^\infty \frac{dx}{\sqrt{x^4+1}}$
45. $\int_0^\infty \frac{dy}{(1+y^2)(1+\tan^{-1}y)}$
46. $\int_{-\infty}^\infty \frac{e^{-y}\,dy}{y^2+1}$

## Explorations

49. *The Infinite Paint Can, or Gabriel's Horn* Consider the region $R$ in the first quadrant bounded above by $y = 1/x$ and on the left by $x = 1$. The region is revolved about the $x$-axis to form an infinite solid as shown in the figure.




(a) **Writing to Learn** Explain how Example 1 shows that the region $R$ has infinite area.

(b) Find the surface area of the solid.

(c) Find the volume of the solid.

(d) Why is Gabriel's horn sometimes described as a can that does not hold enough paint to cover its own outside surface?

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