Connecting the graphs of a function and its derivative
Lesson Objectives

1. From a graph of a function, sketch its derivative

2. From a graph of a derivative, graph an original function
Example #1

\[ f(x) = x^2 - 4x - 5 \]

derivative of \( f(x) \)
Example #1

Points to note:

(1) the f(x) has a minimum at x=2 and the derivative has an x-intercept at x=2
(2) the f(x) decreases on (-∞,2) and the derivative has negative values on (-∞,2)
(3) the f(x) increases on (2,+∞) and the derivative has positive values on (2,+∞)
(4) the f(x) changes from decrease to increase at the min while the derivative values change from negative to positive
Example #1

Points to note:

(5) the function is concave up and the derivative is increasing

(6) The second derivative of $f(x)$ is positive everywhere
Example #2

\[ f(x) = (x-1)(x+2)(x+4) \]

derivative of \( f(x) \)
Example #2

- $f(x)$ has a max. at $x = -3.1$ and $f'(x)$ has an $x$-intercept at $x = -3.1$.
- $f(x)$ has a min. at $x = -0.2$ and $f'(x)$ has a root at $-0.2$.
- $f(x)$ increases on $(-\infty, -3.1)$ & $(-0.2, \infty)$ and on the same intervals, $f'(x)$ has positive values.
- $f(x)$ decreases on $(-3.1, -0.2)$ and on the same interval, $f'(x)$ has negative values.
- At the max $(x = -3.1)$, $f(x)$ changes from increasing to decreasing $\Rightarrow$ the derivative changes from positive values to negative values.
- At a the min $(x = -0.2)$, $f(x)$ changes from decreasing to increasing $\Rightarrow$ the derivative changes from negative to positive.
Example #2

- At the max \((x = -3.1)\), \(f(x)\) changes from increasing to decreasing \(\Rightarrow\) the derivative changes from positive values to negative values
- At a the min \((x = -0.2)\), \(f(x)\) changes from decreasing to increasing \(\Rightarrow\) the derivative changes from negative to positive
- \(f(x)\) is concave down on \((-\infty, -1.67)\) while \(f'(x)\) decreases on \((-\infty, -1.67)\)
- \(f(x)\) is concave up on \((-1.67, \infty)\) while \(f'(x)\) increases on \((-1.67, \infty)\)
- The concavity of \(f(x)\) changes from CD to CU at \(x = -1.67\), so \(f(x)\) has an inflection point at \(x = -1.67\), while the derivative has a min. at \(x = -1.67\)
Let’s try to match a function with its derivative.
• To further visualize the relationship between the graph of a function and the graph of its derivative function, we can run through some exercises wherein we are given the graph of a function \(\rightarrow\) can we draw a graph of the derivative and vice versa
Matching Function Graphs and Their Derivative Graphs
Matching Function Graphs and Their Derivative Graphs - Answer
Matching Function Graphs and Their Derivative Graphs – Working Backwards
Matching Function Graphs and Their Derivative Graphs – Working Backwards
Continuity and Differentiability

• Graph the derivatives of the following function:
Continuity and Differentiability

• Graph the derivatives of the following function:

• Continuous functions are non-differentiable under the following conditions:
  ▪ The function has a “corner”
  ▪ The function has a “cusp”
  ▪ The function has a vertical tangent

• This non-differentiability can be seen in that the graph of the derivative has a discontinuity in it!
• Graph the function given its derivative:

\[ f'(x) = \frac{-2x(1-x^2)}{|1-x^2|} \]
Continuity and Differentiability

- Graph the function given its derivative: