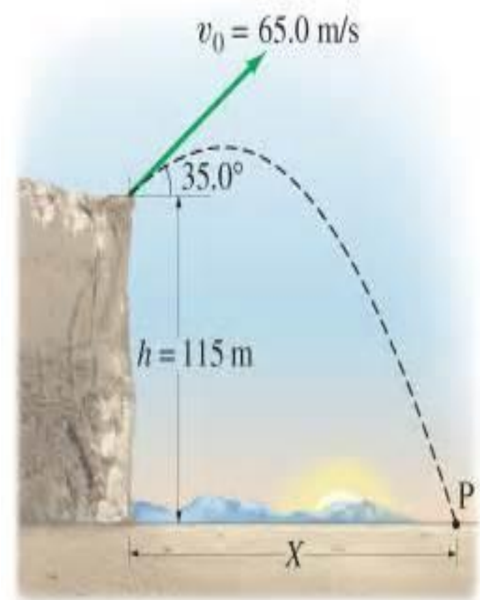
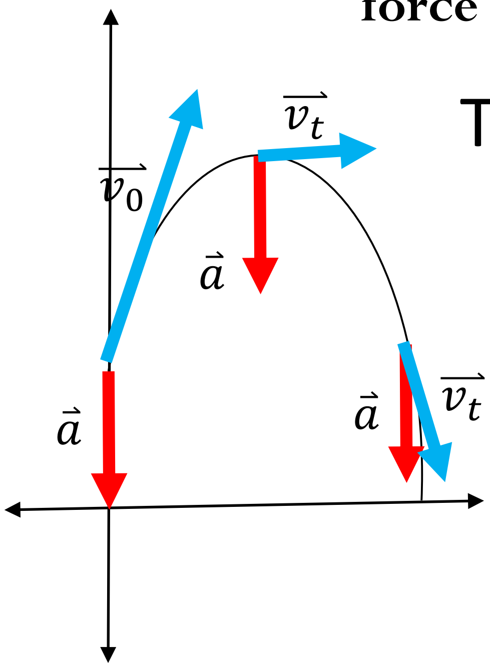
Ideal Projectile Motion

- To derive equations for *ideal* projectile motion, we assume that the only force acting on the projectile during its flight is the constant force of gravity, which always points straight down. In practice, the ground moves beneath the projectile as the earth turns, the air creates frictional force that varies with the projectile's speed and altitude, and the force of gravity changes magnitude and direction as the projectile moves along. All this must be taken into account by applying corrections to the predictions of the ideal equations we are about to derive.

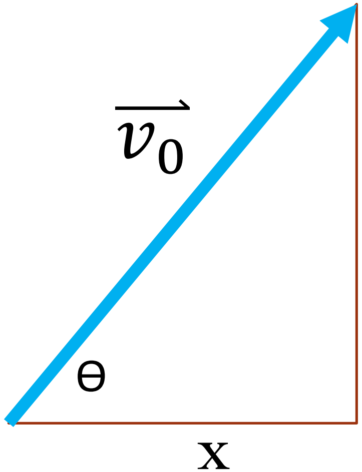


Newton's second law of motion says that $F = m\vec{a}$. If the force is solely the gravitational force $-mg\vec{j}$, then $m\vec{a} = -mg\vec{j}$.



Therefore $\vec{a} = -g\vec{j} = \langle 0, -g \rangle$.

From here, solve for $\vec{v}$



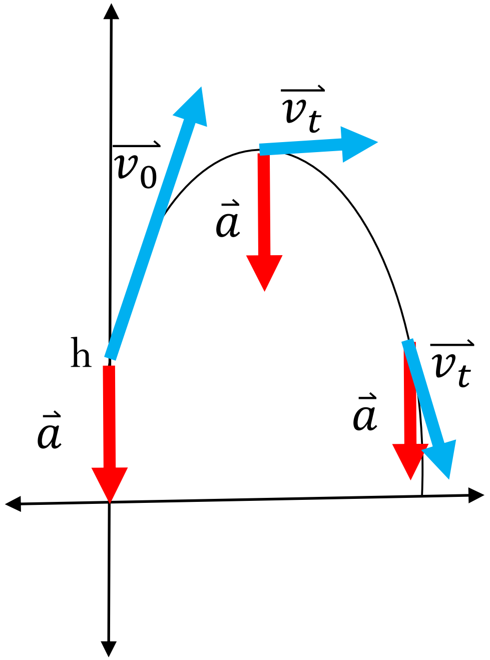
$\cos\theta = \frac{x}{|\overrightarrow{v_0}|}$ $x = |\overrightarrow{v_0}|\cos\theta$

$\sin\theta = \frac{y}{|\overrightarrow{v_0}|}$ $y = |\overrightarrow{v_0}|\sin\theta$

initial velocity $\vec{v}(0) =$

$\left\langle |\overrightarrow{v_0}|\cos\theta, |\overrightarrow{v_0}|\sin\theta \right\rangle$

From here, solve for $\vec{r}$



$$\vec{v} = \langle |\overrightarrow{v_0}| \cos\theta, |\overrightarrow{v_0}| \sin\theta - gt \rangle$$

Initial position: $\overrightarrow{r_0} = \langle 0, h \rangle$

$$\vec{r}(t) = \left\langle (v_0 \cos\theta)t, h + (v_0 \sin\theta)t - \frac{1}{2}gt^2 \right\rangle$$

Gravity = 32 ft/sec^2
or 9.8 m/ sec^2

Example:

A baseball is hit 3 feet above ground at 100 feet per second and at an angle of $\frac{\pi}{4}$ with respect to the ground.

a. Find the maximum height reached by the baseball.

b. Will it clear a 10 foot high fence located 300 feet from home plate?