Integration

AP Exam Review

Derivates vs Integrals

Derivatives are about <u>change.</u>

Velocity is <u>change</u> in position over time.
Acceleration is <u>change</u> in velocity over time.

Integrals are about <u>accumulation.</u>

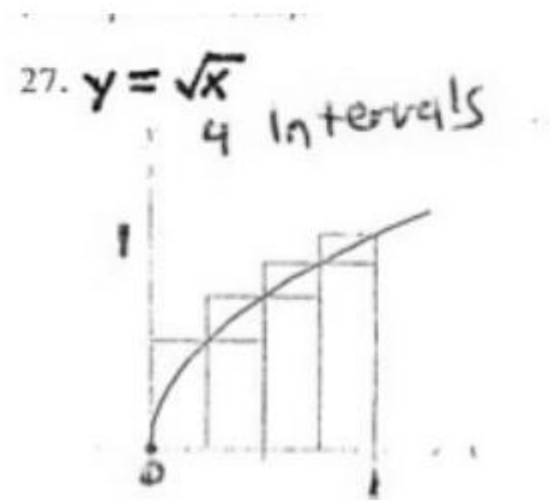
Position is <u>accumulated</u> distance due to velocity.
Money in the bank is <u>accumulated</u> interest.
Bacteria in a petri dish is <u>accumulated</u> over time.

If you do not know if a problem is a derivative or an integral problem, ask yourself if it is asking about rates of change or how something is accumulating.

Riemann Sums

Integration is a method to find the area under a curve.

$$Area = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$$



LRAM

RRAM

Midpoint
– midpoint of x–interval, not the geometric midpoint

Trapezoidal

$$A = \frac{1}{2}h\left(b_0 + 2b_1 + 2b_2 + ... + 2b_{n-1} + b_n\right)$$

$h =$ width of partition

$b =$ value of function at the specific $x -$ value

If intervals are not uniform width, rectangles and trapezoids must be calculated individually

AP Tips

- Don’t confuse upper and lower sums with left and right hand methods. Upper sums refer to rectangles that extend above the curve, while lower sums involve those that lie inside the curve.
- The AP exam often will specify a set number of intervals and give you a table of values to utilize in the use of the Trapezoidal Rule.

Estimate the area under the curve $f(x) = 2x^2$ from $x = 0$ to 12 using LRAM, RRAM, and MRAM and three equal-sized partitions.

Use trapezoidal approximation to estimate how far the train traveled in 8 hours.

Time (hours)	0	1	3	4	7	8
Velocity (m/hr)	0	12	10	5	6	2

You do not need to simplify the expression to receive full credit.

$$\int_2^5 x^2 - 5\,dx$$

Is called a definite integral.
We can evaluate it and get a numerical answer.

$$\int x^2 - 5\,dx$$

Is called an indefinite integral.
Its solution is the set of all possible antiderivatives.

Don't forget the + C!!!

The fundamental Theorem of Calculus

- Part 1: If a function f is continuous on the closed interval [a,b] and F is the antiderivative of f on the interval, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

A little trickier

Find $F(4)$

given $F(x) = \int x^2 e^{2x-1} \cos x \ln x \; dx$ $\qquad F(1) = 2$

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

Useful whenever all you need is the value at a specific point and finding the antiderivative is impractical.

Derivative of an Integral is the original function plus the chain rule

$$\frac{d}{dx}F(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_a^u f(t)\,dt = f(u)\frac{du}{dx}$$

$$Example \qquad \frac{d}{dx}\int_0^{5x} t^3 + 3t - 1\,dt =$$

Understand the difference between finding an integral, which is a number (positive, negative, or zero), and finding the area under the curve, which will always be a positive number.

Know how to graph an absolute value function so that, when you find the area under the curve, you know to split it into two or more integrals, splitting the intervals used for integration at the function's zeros.

Don't forget about symmetry.
If the function is even, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.
If the function is odd, $\int_{-a}^{a} f(x)dx = 0$.

Integration by Hand

- Review basic rules.

reverse power rule, trig rules, exponential rule, inverse function (1/x) rule.

uSubstitution:

$$\int (x^2+5)^3(2x)\,dx =$$

You can introduce a constant as long as you do it in such a way that the problem remains the same; you cannot introduce a variable (like multiplying and dividing by x).

More U-Sub

- $\int \frac{x^2+1}{x^3+3x}\,dx$

Inverse Trig Integrals

$$\int \frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x)+c$$
$$\int \frac{1}{1+x^2} = \tan^{-1}(x)+c$$
$$\int \frac{1}{|x|\sqrt{x^2-1}} = \sec^{-1}(x)+$$

1. $\displaystyle\int \frac{4}{4x^2+1}\,dx$

What inverse trig function looks like the best fit?

Get the 1 in the bottom

Factor constants outside the integral

Write as (something)^2

Use u-Substitution with inverse trig formula

Integration by Parts

$$\int x^2 \ln x dx$$

Partial Fractions

1. If top degree >= bottom degree, then divide bottom into the top

2. Factor the bottom

3. Break into fractions using each factor (factors with multiplicity > 1 appear multiple times)

4. Multiply through by the common denominator

5. Find convenient values for *x* that make groups disappear

6. Find A, B, …

7. Rewrite and solve the integral

Solve $\int \frac{1}{x^2 - 5x + 6} dx$

Compare the following:

- $\int \frac{3}{x^2+9}dx \qquad \int \frac{3x}{x^2+9}dx \qquad \int \frac{3x^2}{x^2+9}dx$

Improper Integrals

$$\int_0^1 \frac{1}{\sqrt[3]{x}}\,\mathrm{dx}$$

Average Value Theorem: If a function is integrable on [a,b], then its average y–value is:

$$Average = \frac{1}{b-a}\int_a^b f(x)\,dx$$

On the exam you are sometimes asked for the average value, the f(c), but you could be asked where it occurs, the c-value in the interval from a to b. Remember to always reread the problem to determine the actual response required.

On the exam, the question may not ask for the term average value; for example, a velocity problem might ask for the average velocity. Depending on what function is given, position or velocity, you have to use different formulas to find average velocity.

If the position function is given: $v_{av} = \frac{s(b)-s(a)}{b-a}$

If the velocity function is given: $v_{av} = \frac{1}{b-a}\int_a^b v(t)dt$