

Integration of Vector-Valued Functions

$$\vec{r} = \langle f(x), g(x) \rangle \quad \int \vec{r} dt = \langle \int f(x) dt, \int g(x) dt \rangle$$

Example:

Given $\vec{v} = \langle \frac{1}{t+1}, 2t \rangle$, find the particle's position vector if $\vec{r}(1) = \langle \ln 2, 0 \rangle$ and find the distance travel from $t = 0$ to $t = 2$.

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To find total distance traveled:

use arc length if given position vector or use $\int |\vec{v}| dt$ if given velocity vector

Example (calculator active):

An object moving along a curve in the xy -plane has position $(x(t), y(t))$ at time t

with $\frac{dx}{dt} = \sin(t^3)$, $\frac{dy}{dt} = \cos(t^2)$. At time $t = 2$, the object is at the position $(1, 4)$.

- (a) Find the acceleration vector for the particle at $t = 2$.
- (b) Write the equation of the tangent line to the curve at the point where $t = 2$.
- (c) Find the speed of the vector at $t = 2$.
- (d) Find the position of the particle at time $t = 1$.