Introduction to Riemann Sums

A car is traveling so that its speed is never decreasing during a 10 second interval. The speed at various points in time is listed below.

Time (Sec)	0	2	4	6	8	10
Speed (ft/sec)	30	36	40	48	54	60

1. What is the best lower estimate for the distance the car traveled in the first 2 seconds? 30(2) = 60 feet

2. What is the best upper estimate for the distance the car traveled in the first 2 seconds? 36(2) = 72 feet

Time (Sec)	0	2	4	6	8	10
Speed (ft/sec)	30	36	40	48	54	60

3. What is the best lower estimate for the total distance traveled during the first 4 seconds?

$$30(2) + 36(2) = 132 \text{ feet}$$

4. What is the best upper estimate for the total distance the car traveled in the first 4 seconds?

$$36(2) + 40(2) = 152 \text{ feet}$$

Time (Sec)	0	2	4	6	8	10
Speed (ft/sec)	30	36	40	48	54	60

5. Continuing this process, what is the best lower estimate for the total distance traveled during the first 10 seconds?

$$30(2) + 36(2) + 40(2) + 48(2) + 54(2) = 416 \text{ feet}$$

6. Continuing this process, what is the best upper estimate for the total distance the car traveled in the first 10 seconds?

$$36(2) + 40(2) + 48(2) + 54(2) + 60(2) = 416 \text{ feet}$$

These estimates are sums of products and are known as **<u>Riemann Sums</u>**.

7. If you choose the lower estimate as your approximation of how far the car traveled, what is the maximum amount your approximation could differ from the actual distance?

60 feet

- In the table below, fill in the missing speeds. You can choose whatever speeds you wish as long as the car never slows down.

Time (Sec)	0	1	2	3	4	5	6	7	8	9	10
Speed (ft/sec)	30		36		40		48		54		60

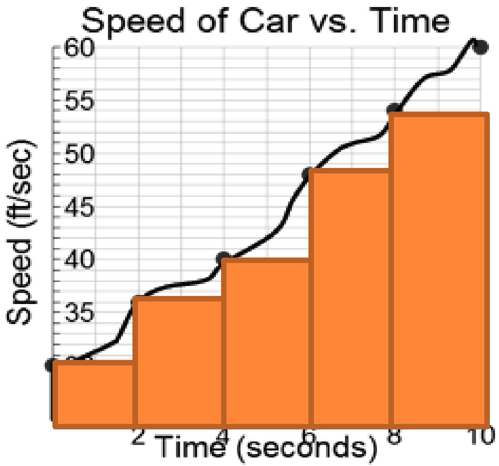
- Repeat the process to find the best lower and upper estimates for how far the car traveled. If you choose the lower estimate, what is the maximum amount your approximation could differ from the actual distance?

30 feet

More points cause the range to decrease.

The graph at the right shows the actual speed of the car at all times. The lower estimate for the first 2 second interval was 2 seconds times 30 ft/sec which equals 60 ft. This can be represented by the rectangle shown where the area of the rectangle gives us the distance traveled.

Draw in rectangles to show how you calculated the lower estimate for each of the other intervals.



Notice how the **left side** of each rectangle touches the graph.

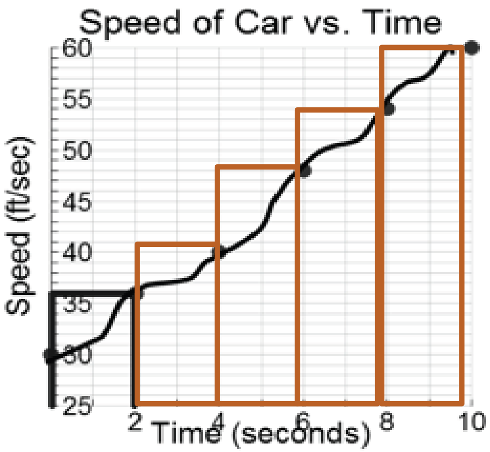
This method of approximating the area under a curve is called the **Left-Hand Rectangular Approximation Method (LRAM).**

Each interval is called a **partition**. The partition width is the difference in the x-values of each partition.

The graph at the right shows the upper estimate for the first interval. (2 sec times 36 ft/sec = 72 ft.) The area of the rectangle is 2 x 36 = 72.

Draw in rectangles to show how you calculated the upper estimate for each of the other intervals.

Notice how the **right side** of each rectangle touches the graph.

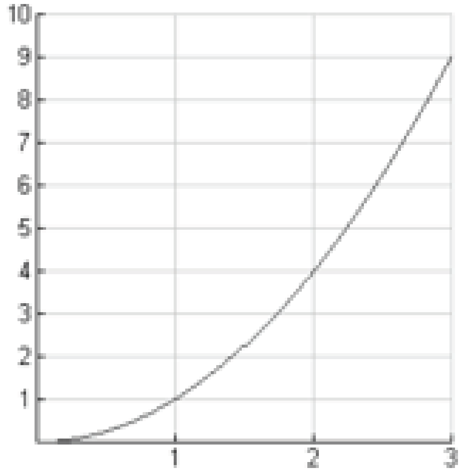


This method of approximating the area under a curve is called the **Right-Hand Rectangular Approximation Method (RRAM).**

The approximations we calculated above are known as **Riemann Sums** (named after Georg Riemann). The maximum error in a Riemann Sum is the difference between the lowest estimate and the highest estimate. The accuracy of the approximation can be improved by increasing the number of rectangles.

EXAMPLE

- Find the area between the x-axis and $y = x^2$ on the interval [0, 3] with partition width of 1.
- A. Left-Hand Approximation (LRAM)



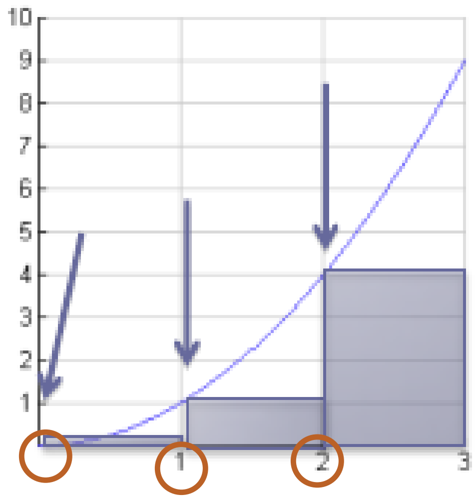
- B. Right-Hand Approximation (RRAM)
- C. Midpoint Approximation (MRAM)

Area Under Curves

Find the area between the x-axis and $y = x^2$ between [0,3] with partition widths of 1.

$$L_3 = 1(0) + 1(1) + 1(4) = 5$$

a) Left-hand approx: (LRAM)



$y(2) = 2^2 = 4$

$y(1) = 1^2 = 1$

$y(0) = 0^2 = 0$

Rectangular
Approximation
Method

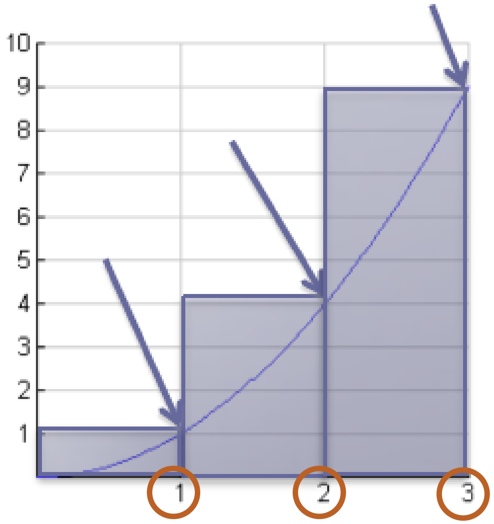
LRAM: The left side of each rectangle touches the graph.

Area Under Curves

Find the area between the x-axis and $y = x^2$ between [0,3] with partition widths of 1.

$R_3 = 1(1) + 1(4) + 1(9) = 14$

b) Right-hand approx: (RRAM)



$y(3) = 3^2 = 9$

$y(2) = 2^2 = 4$

$y(1) = 1^2 = 1$

RRAM: The left side of each rectangle touches the graph.

EXAMPLE

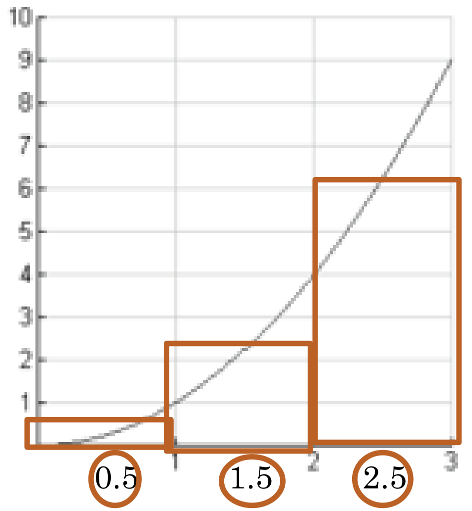
Find the area between the x-axis and $y = x^2$ on the interval [0, 3] with partition width of 1.

C. Midpoint Approximation (MRAM)

$y(2.5) = 2.5^2 = 6.25$

$y(1.5) = 1.5^2 = 2.25$

$y(0.5) = 0.5^2 = 0.25$

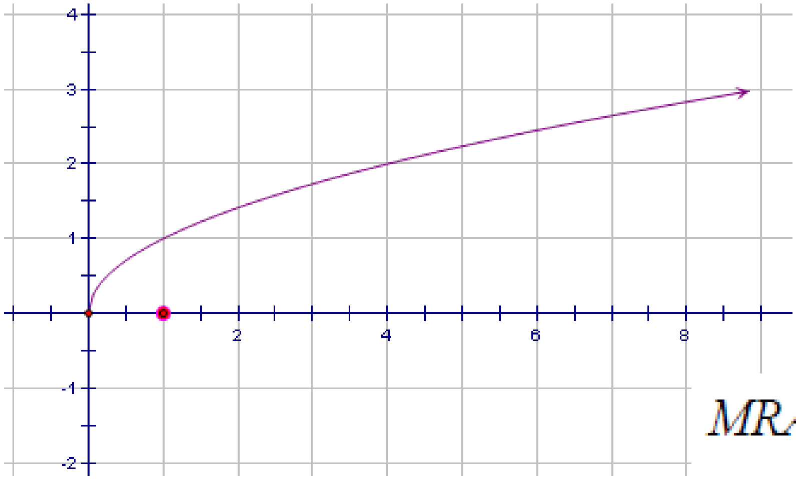


$M_3 = 1(0.25) + 1(2.25) + 1(6.25) = 8.75$

You Try

- Find the area under the curve $y = \sqrt{x}$ from [0, 9] using 3 equal partitions.
- Use LRAM, RRAM and MRAM

$$\text{Width} = \frac{b-a}{n} = \frac{9-0}{3} = 3$$



$$LRAM = 3(0) + 3\left(\sqrt{3}\right) + 3\left(\sqrt{6}\right) \approx 12.545$$

$$RRAM = 3\left(\sqrt{3}\right) + 3\left(\sqrt{6}\right) + 3\left(\sqrt{9}\right) \approx 21.545$$

$$MRAM = 3\left(\sqrt{1.5}\right) + 3\left(\sqrt{4.5}\right) + 3\left(\sqrt{7.5}\right) \approx 18.254$$

On a non-multiple choice question, you must show all of the work above to demonstrate your understanding and earn full credit.