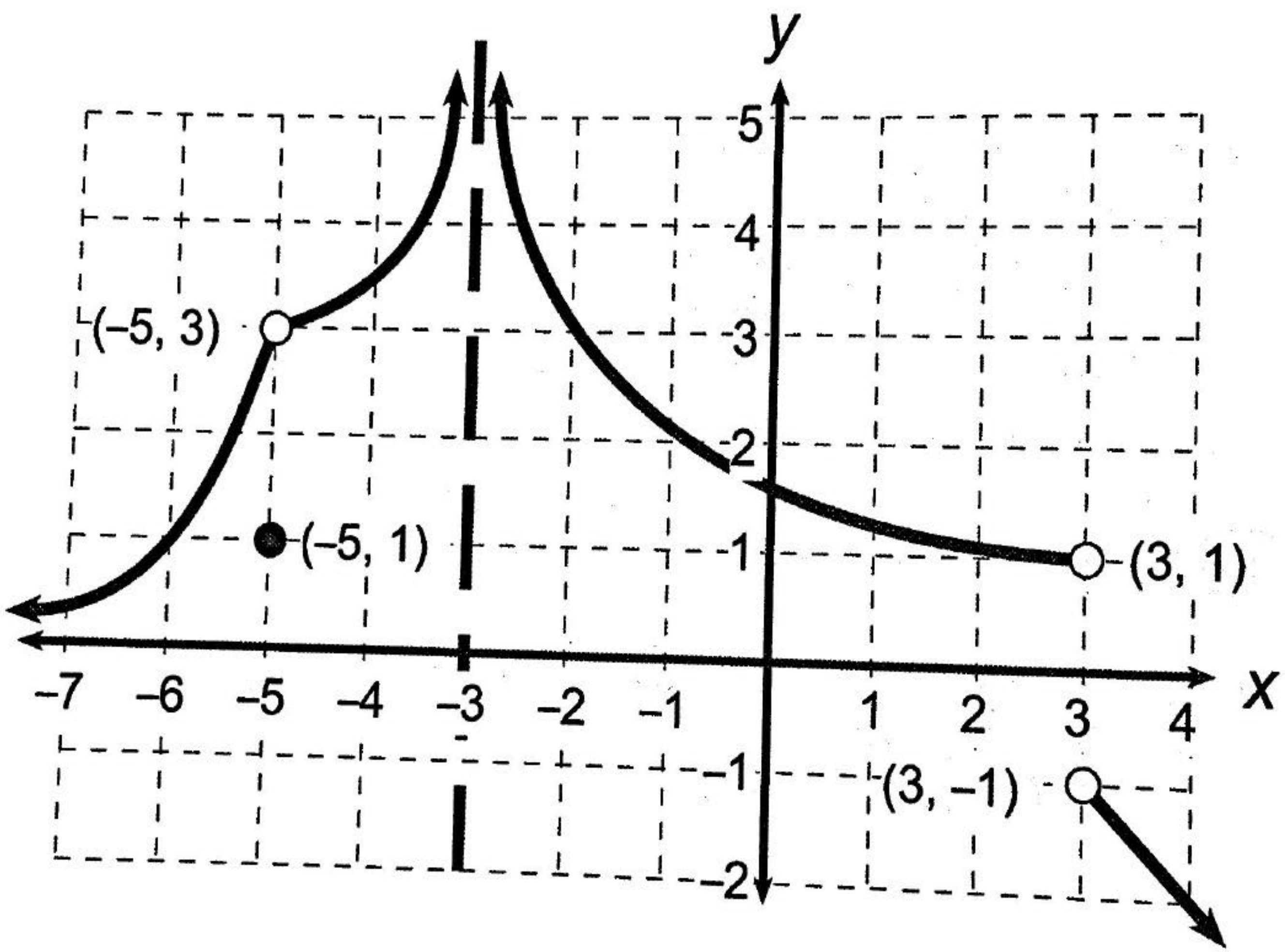
Limits and Derivatives

AP Exam Review

Objectives

- Understand limits, including asymptotic and unbounded behavior.
- Calculate limits using algebra.
- Estimate limits from graphs and data.
- Understand one-sided limits.
- Understand continuity in terms of limits.
- Understand the concept of the derivative-numerically(from tables),graphically, and analytically-and be able to use the limit definition.
- Understand instantaneous versus average rate of change.
- Find and use tangent lines to a curve at a given point.
- Be able to apply differentiation rules.

Graphically Evaluate Limits



1. $\lim_{x \to -5} f(x) =$

2. $\lim_{x \to -3} f(x) =$

3. $\lim_{x \to 3+} f(x) =$

4. $\lim_{x \to 3-} f(x) =$

5. $\lim_{x \to 3} f(x) =$

Strategies for Solving a Limit Problem

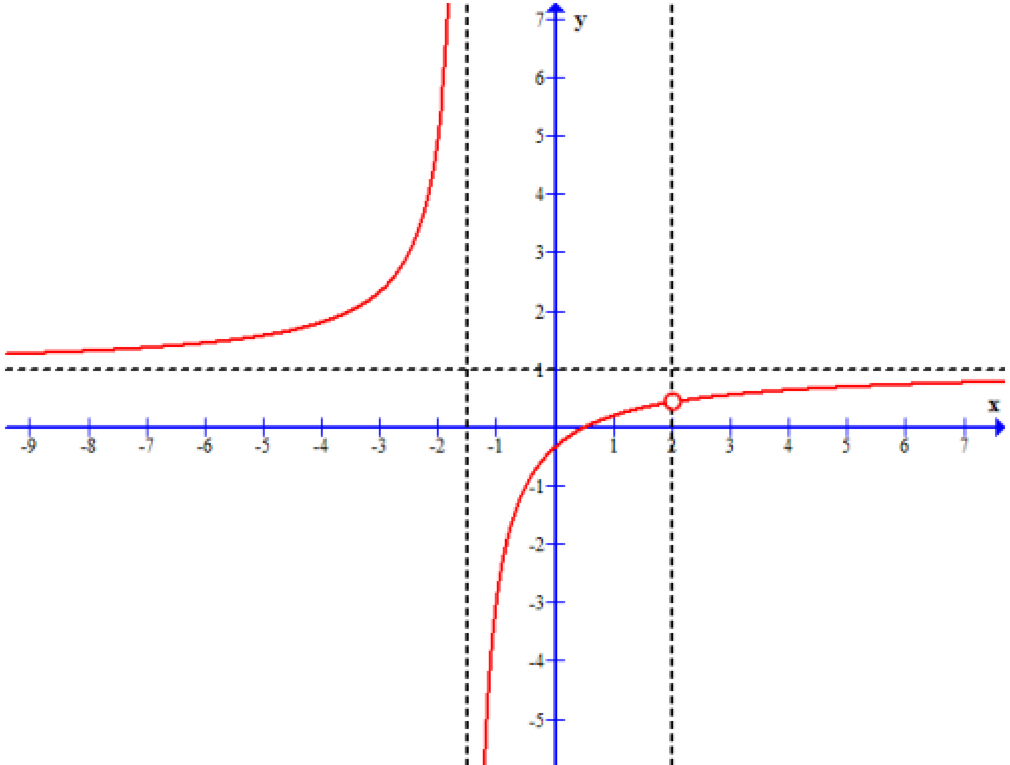
1. Always begin with direct substitution.
2. Try to simplify and cancel.
3. Try to rationalize the denominator or numerator.
4. Try using your calculator to interpret the data.
   - Enter the function in y=.
   - Go to the table set up to fix your Δ to a small value like 1/10
   - Look at the y-values as the x-values get close from both the left and the right.
5. Two special limits you will want to know:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \qquad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

6. L'Hopital can be used for the indeterminate forms: $\frac{0}{0}$ or $\frac{\infty}{\infty}$

Infinite Limits vs. Limits at Infinity

- Infinite limits: occur when y approaches infinity as x approaches a number. Visually the graph approaches a VERTICAL ASYMPTOTE.
- Limits at Infinity: occur when y approaches a number as x approaches infinity. Visually the graph approaches a HORIZONTAL ASYMPTOTE.



$$\lim_{x \to -1.5-} f(x) = \infty$$

$$\lim_{x \to -1.5+} f(x) = -\infty$$

$$\lim_{x \to -1.5-} f(x) = DNE$$

$$\lim_{x \to -\infty} f(x) = 1$$

$$\lim_{x \to \infty} f(x) = 1$$

Rational Functions: HA, VA and holes

- $f(x) = \frac{1}{x-2}$ $\quad f(x) = \frac{(x-3)(x+2)}{x-3}$ $\quad f(x) = \frac{3x^2+5x}{x^2}$

Continuity

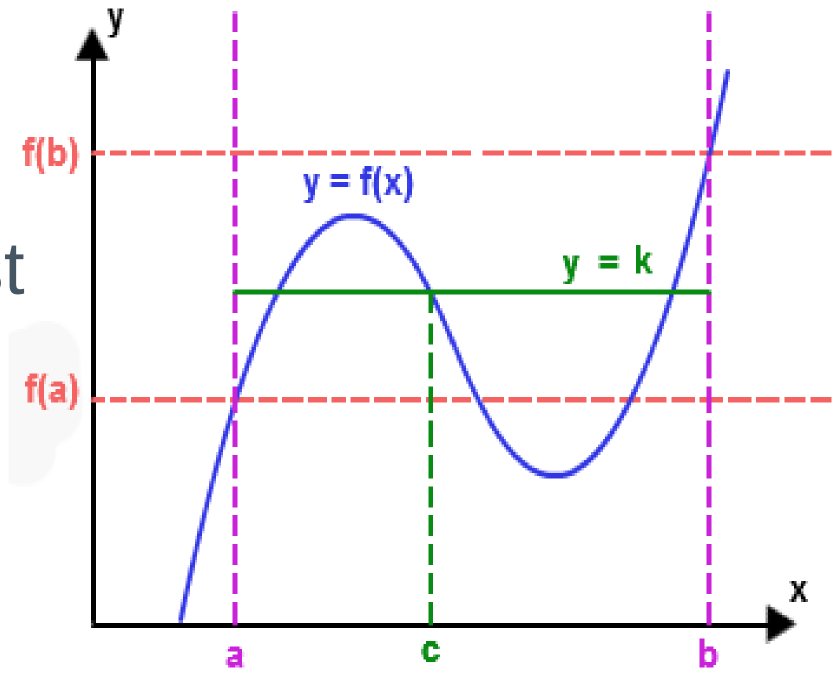
Definition: A function f is continuous at c if the following three conditions exist:

1. f(c) is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

A question on the 2003 exam required students to show all 3 components of this definition.

Intermediate Value Theorem

- If a function f is continuous on a closed interval [a,b] and if there is some y-value (called k) between f(a) and f(b), then there has to be at least one x-value between a and b (called c) such that f(c) = k.
- This is an existence theorem. It guarantees that such a number exists, but does not provide a method for finding the number.



Function f is continuous on $[0,4]$. Given the following values, what is the minimum number of times $f(x)=8$?

Justify.

x	0	1	2	3	4
$f(x)$	10	6	2	7	9

<u>Solution</u>

$f(x)=8$ at least 2 times.

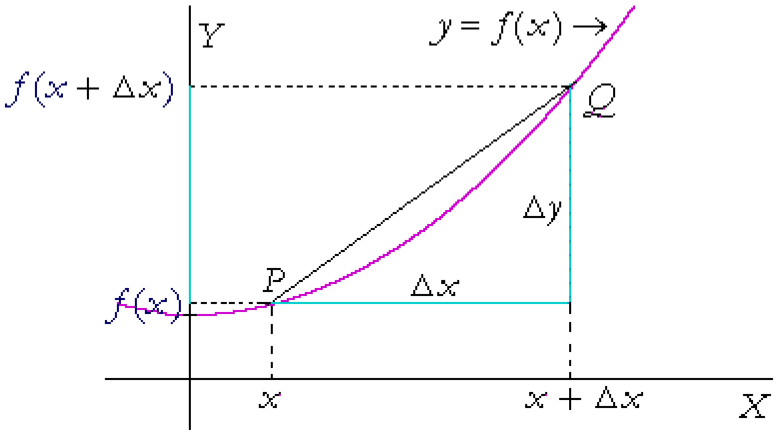
$f(x)$ is continuous on $[0,1]$ therefore $f(x)$ takes on all values between 10 and 6 by the <u>Intermediate Value Theorem</u>

$f(x)$ is continuous on $[3,4]$ therefore $f(x)$ takes on all values between 7 and 9 by the <u>Intermediate Value Theorem</u>

Derivative of a Function

- The derivative of a function is the rate of change of the function.
- The derivative at point x is equal to the slope of the tangent line at the point (x, f(x)).
- The derivative of f at a point is given by the following limit, provided the limit exists. This form is called the **difference quotient.**

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$



Alternate form:

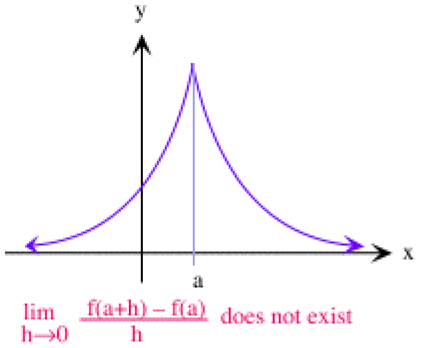
$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

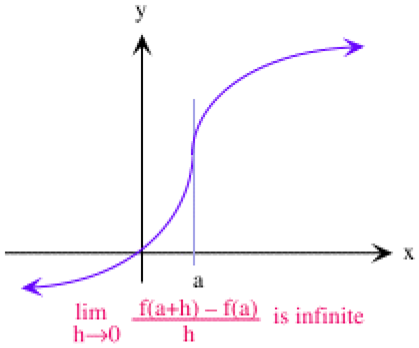
- You need to know how to find the derivative using the limit process and to be able to recognize the difference quotient in its limit form.
- You need to be able to apply the Constant Rule, Power Rule, Product Rule, Quotient Rule, and Chain Rule. Need to know all 24 derivative rules on p.5 of blue sheets.
- You should be able to recognize various notations for derivatives.

$$f'(x), \frac{dy}{dx}, y', \frac{d}{dx}[f(x)] \qquad\qquad f''(x), \frac{d^2y}{dx^2},$$

Differentiability

- If a function is differentiable at x = c, it is also continuous at x = c. The converse is not true. Continuity does not imply differentiability.
- A function is not differentiable at a point if one of the following is true:
  1. It is discontinuous at the point.
  2. A sharp turn(corner or cusp) occurs.
  3. There is a vertical tangent line at the point.





Implicit Differentiation

- Use when it is difficult to solve an equation for y.
- Example: $x^2 - 2y^3 + 4y = 2$

Logarithmic Differentiation

- Example: $y = \frac{2x(x-2)^2}{\sqrt{x^2+1}}$

$$y = x^{\sin x}$$