

Part II. Use the process outlined in Part I to find a limit expression for the area of each region bounded by the given function and the x-axis on [a, b].

Function, [a, b]

Limit expression of the Riemann sum

1. $f(x) = 3x$ on $[0, 2]$ $\Delta x = \frac{2}{n}$, $x_i = 0 + \frac{2i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n 3\left(\frac{2i}{n}\right) \frac{2}{n}$ ①

2. $f(x) = x^2$ on $[0, 3]$ $\Delta x = \frac{3}{n}$, $x_i = 0 + \frac{3i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{3i}{n}\right)^2 \frac{3}{n}$ ②

3. $f(x) = 2x^2$ on $[1, 3]$ $\Delta x = \frac{3-1}{n}$, $x_i = 1 + \frac{2i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n 2\left(1 + \frac{2i}{n}\right)^2 \frac{2}{n}$ ③

4. $f(x) = 1 - x^2$ on $[-1, 1]$ $\Delta x = \frac{1-(-1)}{n}$, $x_i = -1 + \frac{2i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[1 - \left(\frac{2i}{n} - 1\right)^2\right] \frac{2}{n}$ ④

5. $f(x) = 2x - x^3$ on $[0, 1]$ $\Delta x = \frac{1-0}{n}$, $x_i = 0 + \frac{i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[2\left(\frac{i}{n}\right) - \left(\frac{i}{n}\right)^3\right] \frac{1}{n}$ ⑤

6. $\int_1^3 (2x+1) dx$ $\Delta x = \frac{3-1}{n}$, $x_i = 1 + \frac{2i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[2\left(1 + \frac{2i}{n}\right) + 1\right] \frac{2}{n}$ ⑥

7. $\int_2^4 (3x^2 - 1) dx$ $\Delta x = \frac{4-2}{n}$, $x_i = 2 + \frac{2i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[3\left(2 + \frac{2i}{n}\right)^2 - 1\right] \frac{2}{n}$ ⑦

8. $\int_{-2}^0 (x+3)^2 dx$ $\Delta x = \frac{0-(-2)}{n}$, $x_i = -2 + \frac{2i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(\frac{2i}{n} - 2\right) + 3\right]^2 \frac{2}{n}$ ⑧

9. $\int_0^\pi \sin x dx$ $\Delta x = \frac{\pi-0}{n}$, $x_i = 0 + \frac{\pi i}{n}$ $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sin\left(\frac{\pi i}{n}\right) \frac{\pi}{n}$ ⑨

Part III. In each of the following problems, translate the Riemann sum into a definite integral.

~~As noted in the previous section...~~

1. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[3\left(\frac{3i}{n} + 2\right) - 8\right] \frac{\Delta x}{n}$ $\int_2^5 (3x - 8) dx$
 $b - 2 = 3$
 $b = 5$

2. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[5\left(\frac{5i}{n} - 2\right)^3 - 1\right] \frac{\Delta x}{n}$ $\int_{-2}^3 (5x^3 - 1) dx$
 $b - 2 = 5$
 $b = 3$

3. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[3\left(\frac{2i}{n} - 2\right) + 4\right] \frac{\Delta x}{n}$ $\int_{-2}^0 (3x + 4) dx$
 $b - 2 = 2$
 $b = 0$

4. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\cos\left(\frac{i\pi}{6n}\right)\right] \frac{\Delta x}{n}$ $\int_0^{\pi/4} (\cos x) dx$
 $b - 0 = \pi$