Unit 4
More with Derivatives

Section 7.4 Logarithmic Differentiation
Section 7.5 Inverse Trigonometric Functions

Find the derivative of y.

- Functions can be simplified using logarithms
- Take the natural log of both sides

$$y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^2}$$

- Then differentiate each side with respect to x.

Logarithmic Differentiation must be used when a function is raised to a functional power.

$y = x^4$

Numerical power

Functional base

$y = 4^{3x}$

Functional power

Numerical base

$y = (\sin x)^x$

Functional power

Functional base

Inverse Trig Functions are angles

$$\sin\frac{\pi}{2} = 1 \qquad \sin^{-1} 1 = \frac{\pi}{2} \quad \text{or} \quad \arcsin(1) = \frac{\pi}{2}$$

What angle has sin equal to 1?

Derivatives of Inverse Trig Functions

$$y = \sin^{-1} x$$

$$\sin y = x$$

$$\frac{d}{dx}(\sin y = x)$$

$$\cos y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{1 - \sin^2 y}} = \frac{\mathrm{d}}{\mathrm{dx}}(\sin^{-1}\mathrm{x})$$

Trig Identity:

$$\cos^2 y + \sin^2 y = 1$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$y = \tan^{-1} x$$

$$1 + \tan^2 y = \sec^2 y$$

$\tan y = x$

$$\frac{d}{dx}(\tan y = x)$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1 + \tan^2 y}$$

$$= \frac{1}{1 + x^2} = \frac{d}{dx}(\tan^{-1} x)$$

$$y = \sec^{-1} x$$

$$1 + \tan^2 y = \sec^2 y$$

Sec y = x

$$\tan^2 y = \sec^2 y - 1$$

$$\frac{d}{dy}(\sec y = x)$$

$$\tan y = \sqrt{\sec^2 y - 1}$$

$$\sec y \tan y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec y \tan y} = \frac{1}{\sec y \sqrt{\sec^2 y - 1}} \boxed{= \frac{1}{x\sqrt{x^2 - 1}} = \frac{d}{dx}(\sec^{-1} x)}$$

Co functions are the same but negative

$$\frac{d}{dx}[\cos^{-1} x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\cot^{-1} x] = \frac{-1}{x^2+1}$$

$$\frac{d}{dx}[\csc^{-1} x] = \frac{-1}{x\sqrt{x^2-1}}$$

Examples: Find y'

$$y = x \tan^{-1} \sqrt{x}$$

$$y = \frac{1}{\sin^{-1} x}$$