$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$ 2) $\frac{dP}{dt} = .0015P(150)\left(1 - \frac{P}{150}\right)$

$= .225P\left(1 - \frac{P}{150}\right)$

Logistic Differential Equations

I. Answer the following questions for the given logistic differential equations.

1. $\frac{dP}{dt} = \frac{P}{3}\left(1 - \frac{P}{10}\right)$ 2. $\frac{dP}{dt} = .0015P\frac{(150 - P)}{150}$ 3. $\frac{dP}{dt} = \frac{0.05P - 0.0005P^2}{.05P} = .05P(1 - .01P)$ $\left(1 - \frac{P}{100}\right)$

a. What is the carrying capacity? 1) 10 2) 150 3) 100

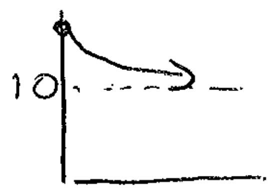
b. What is k? 1) $\frac{1}{3}$ 2) .225 3) .05

c. For what value of P is the population growing the fastest? 1) 5 2) 75 3) 50

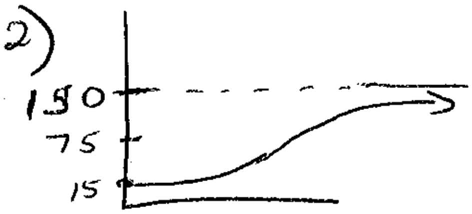
d. If P(0)= 15, $\lim_{t\to\infty} P(t) =$ 1) 10 2) 150 3) 100

d. Sketch the graph of the solution of $\frac{dP}{dt}$ with initial value P(0) = 15. Include horizontal asymptotes and inflection points.

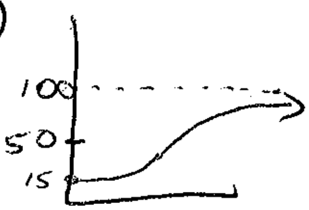
1)



2)



3)



C.C.

II. A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were known to be in the preserve in 1970. The population can be modeled with a logistic differential equation with k = 0.1. Let time be measured in years.

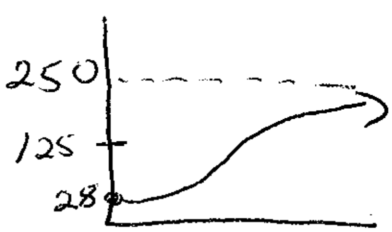
a. Find a logistic differential equation to represent the gorillas' population. $\frac{dP}{dt} = .1P\left(1 - \frac{P}{250}\right)$

b. Sketch a graph of P(t).

c. Find P(t).

d. How long will it take for the gorilla population to reach the carrying capacity of the preserve? (Hint: P cannot equal the carrying capacity, so use a decimal that will round up to the carrying capacity) $A = \frac{250 - 28}{28}$

b)



c) $P(t) = \frac{250}{Ae^{-.1t} + 1} = \frac{250}{\frac{111}{14}e^{-.1t} + 1}$

$\frac{28}{1} = \frac{250}{A + 1}$

$28(A+1) = \frac{250}{28} - 1 = \frac{111}{14}$

$P(t) = \frac{1000}{Ae^{-.8t} + 1}$

$249.5 = \frac{250}{\frac{111}{14}e^{-.1t} + 1}$

$t \approx 83$ yrs