More on Functions

Composition

$f\left(g\left(x\right)\right)$ or $f ⃘ g$, combining two functions to get a new function

Examples: Given $f\left(x\right)= x^{2}$ and $g\left(x\right)=3x-2$ ,

find $f(g\left(x\right))$ and $f(g\left(3\right))$ then find $g(f\left(x\right))$ and $g(f\left(3\right))$



Example: Given

 Find $f(f(-3))$

The domain of the composition is the intersection of the inner function’s domain and the composition’s domain.

Try: $f\left(x\right)=\frac{1}{x^{2}-3}$ and $g\left(x\right)=\sqrt{x-2}$ . Find $f ⃘ g$ and its domain.

Inverse functions

To find the inverse of $f\left(x\right), f^{-1}\left(x\right),$ switch x and y, then solve for y.$ f^{-1}\left(f(x\right))= f( f^{-1}\left(x\right))=x.$

Examples: Find each inverse and compare the graphs of $f\left(x\right) and f^{-1}\left(x\right).$

1. $f\left(x\right)= x^{3}-1$ 2. $y=\frac{2x+1}{x+3}$

Transformations

Given parent function f(x),

* f(x +a) horizontally shifts a spaces to the left
* f(x – a) horizontally shifts a spaces to the right
* f(x) +a horizontally shifts a spaces up
* f(x) – a horizontally shifts a spaces down
* f(-x) reflects across the y-axis
* – f(x) reflects across the x-axis
* c f(x) stretches the graph vertically by a factor of c (example: 2f(x) is twice as tall)
* f(cx) compresses the graph horizontally by a factor of c (ex. f(2x) is half as wide)

Horizontal transformations are the opposite of what common sense tells you.

With combinations of horizontal transformations, deal with add/subt (shifts), then multiplication (stretch/flips).

Examples: Graph the following:

1. $y=x^{2}-9 $

2. $y=4-x^{2}$

3. $y=2+\sqrt{x-1}$

4. $y=-\sqrt{-x}$

5. $y=\frac{1}{x-2}$

6. $y=2\sqrt{3-x}$