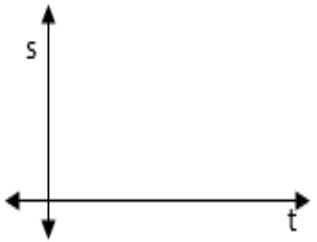
Motion along a Curve

Chapter 13: Vector-Valued Functions

Review of position, velocity, and acceleration

Graphed on the position/time graph



s(t) = Position at time t

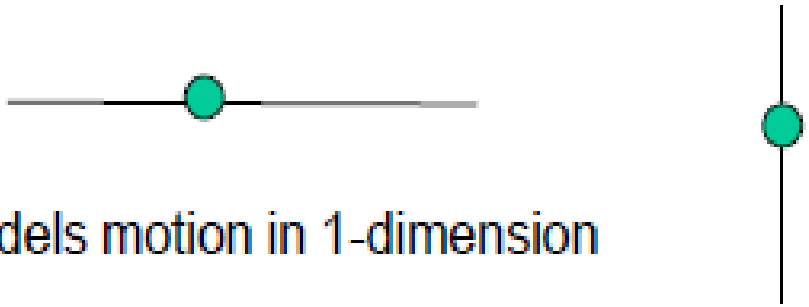
v(t) = Velocity at time t

a(t) = Acceleration at time t

$$v(t) = \frac{ds}{dt}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$|v(t)|$ = Speed



Models motion in 1-dimension

Position, velocity, and acceleration in 2-dimensional space

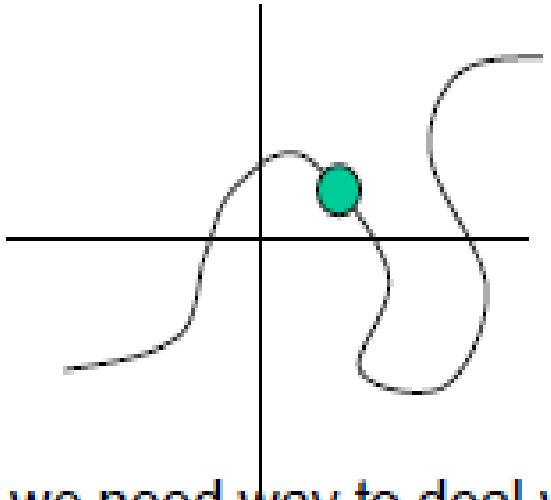
r(t) = Position at time t

v(t) = Velocity at time t

a(t) = Acceleration at time t

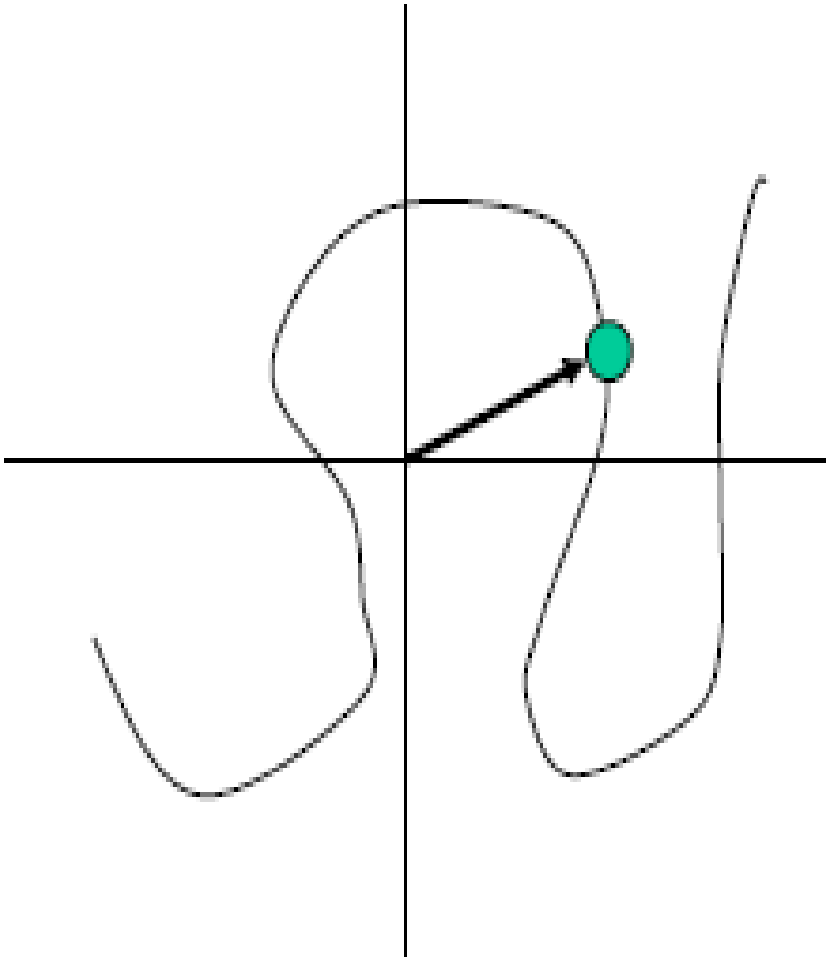
$$v(t) = \frac{dr}{dt}$$

$$a(t) = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

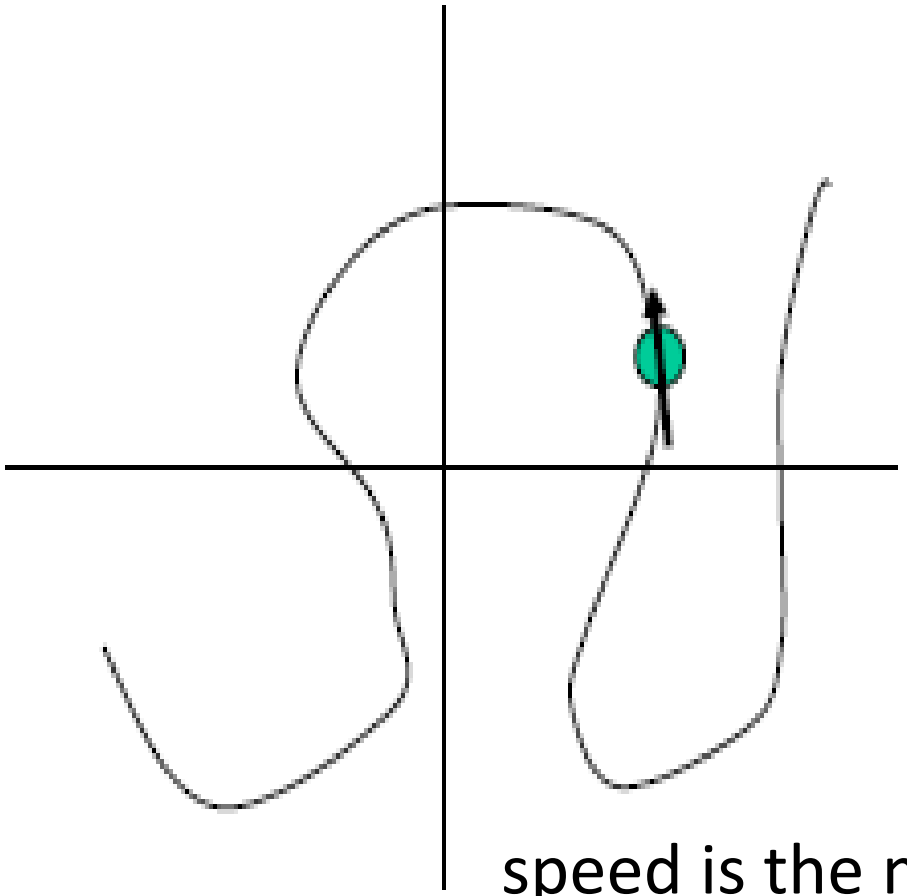


Same idea, but we need way to deal with 2-dimensions

Enter Vectors



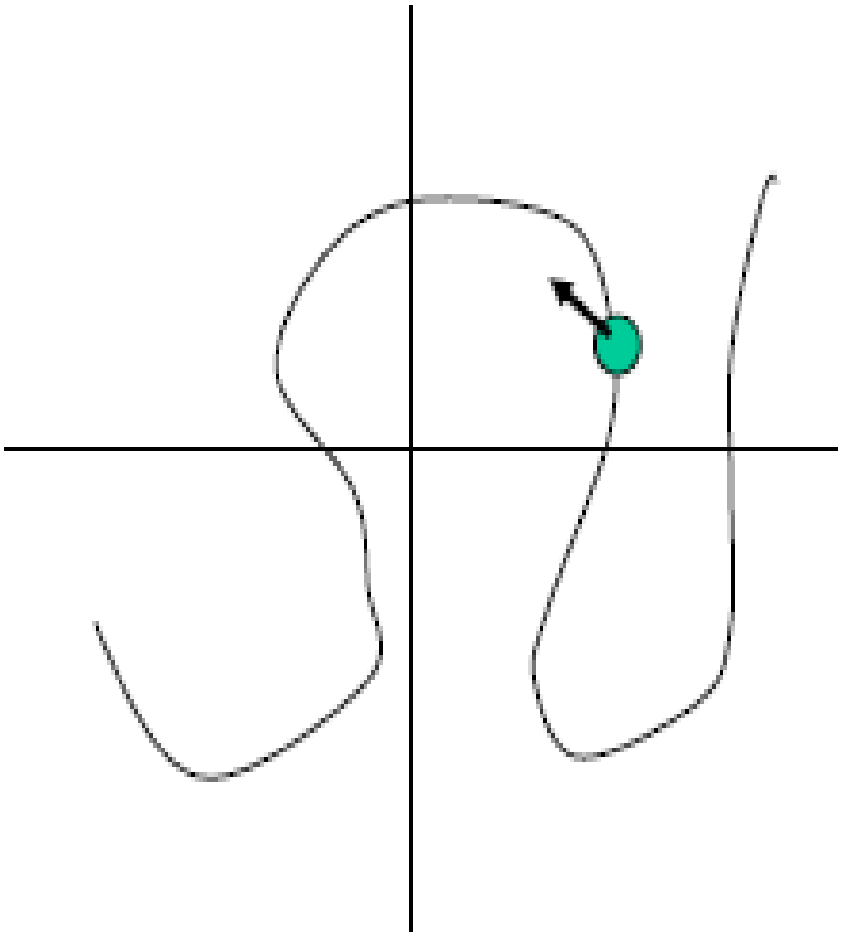
Position vector – position of object at time *t*, relative to the origin.



Velocity vector - direction and speed of object at time t.

speed is the magnitude of velocity = $|\vec{v}|$

direction is the unit vector = $\dfrac{\vec{v}}{|\vec{v}|}$



Acceleration vector – how speed and direction are changing at time *t*.

Example 1:

Given the position vector $\langle 3\cos t, 3\sin t\rangle$, find the velocity and acceleration vectors. Then find velocity, acceleration, speed, and direction of motion at $= \frac{\pi}{4}$.

Example 2:

Given the position vector $\langle 2t^3 - 3t^2, t^3 - 12t \rangle$,

a. Write the equation of the tangent line when $t = -1$.

b. Find coordinates of each point on the path where the particle stops forward/backward movement.

c. When is the particle at rest?

Example 3 (no calculator):

A particle moves in the xy-plane so that $x = \sqrt{3} - 4\cos t$ and $y = 1 - 2\sin t$, where $0 \leq t \leq 2\pi$. The path of the particle intersects the x-axis twice. Write an expression that represents the distance traveled by the particle between the two x-intercepts. Do not evaluate.