**Optimization Practice (4.4)**

1. If 40 passengers hire a special car on a train, they will be charged \$8 each. This fare will be reduced by \$.10 each passenger, for each person in addition to these 40. What number of passengers will produce the maximum profit for the railroad?

Profit = passenger · fare

$= (40+x)(8-.10)$

$P = 320 + 8x - 4x - .10x^2$

$P' = 4 - .20x = 0$

$4 = .20x$

$20 = x$ max

$P'' = -.20$ CD

40 passengers will produce maximum profit

2. A fruit grower estimates that if he harvests his crop of oranges now, he will get 100 pounds per tree, which he can sell for \$.25 per pound. For each week he waits, he estimates that the crop will increase by 10 lb. per tree, but the price will decrease by \$.01 per week. When should he pick the oranges to obtain the maximum profit? What would his profit be at this time?

Profit = lbs per tree · price

$= (100+10x)(.25-.01x)$

$P = 25 + 2.5x - 1x - .01x^2$

$P' = 1.5 - .02x = 0$

$.02x = 1.5$

$x = 75$ max

$P'' = -.2$ CD

He should pick oranges in 7½ wks to maximize profit

His profit at that time will be \$30.63 per tree

3. A rectangular box with a square base and a cover is to be built to contain 640 cubic feet. If the cost per square foot for the bottom is \$15 and for the top and the sides is \$10, what is the minimum cost of the constructed box?

$V = s^2h = 640\ ft^3 \Rightarrow h = \frac{640}{s^2}$

$cost = s^2 15 + s^2 10 + sh \cdot 4 \cdot 10$

$cost = 25s^2 + 40s\left(\frac{640}{s^2}\right)$

$Cost = 25s^2 + \frac{25600}{s} s^{-1}$

$C' = 50s - \frac{25600}{s^2} = \frac{50s^3 - 25600}{s^2} = 0$

$50s^3 = 25600$

$s^3 = 512$

$s = 8$

Min Cost is \$4800

4. A tinsmith wishes to make an open box from a square piece of tin which measures 8" by 8". To accomplish this task, he proposes to cut equal square pieces from each corner of the tin and fold up the tin to form sides. Determine the sides of the squares to be cut from the corners so that the box will have the greatest possible volume. What is this volume?

$V = (8-2x)^2 x$

$V = 64x - 32x^2 + 4x^3$

$V' = 64 - 64x + 12x^2 = 0$

max $x = \frac{4}{3}$, $\cancel{4}$

$V'' = -64 + 24x$

$V''(\tfrac{4}{3}) = -64 + 32 = -32$ CD

Should cut squares of $\frac{4}{3} \times \frac{4}{3}$ in to get max volume of $37.926\ in^3$

5. Find two numbers whose sum is 48 and whose product is to be a maximum.

$x + y = 48 \Rightarrow x = 48 - y$

$P = x \cdot y$

$P = (48-y)y$

$P = 48y - y^2$

$P' = 48 - 2y = 0$

$y = 24$ max

$x = 24$

$P'' = -2$ CD

The 2 # whose sum is 48 and whose product is a max are 24 and 24.

6. Suppose that a rancher has 1000 feet of fencing available to make a rectangular corral. A barn will form one side of the corral so no fencing will be needed there. What dimensions will give the maximum area?

$2x + y = 1000$

$A = xy = x(1000 - 2x)$

$A = 1000x - 2x^2$

$A' = 1000 - 4x = 0$

$x = 250$

$y = 500$

corral should be 250 by 500 ft to create largest area

7. Allen Rent-A-TV derives an average profit of \$15 per customer if it services 1000 customers or less. If it services over 1000 customers, the profit decrease per customer by \$.01 for each customer over 1000. How many customers will give the maximum profit?

Profit = cust · charge

$= (1000+x)(15-.01x)$

$= 15000 + 15x - 10x - .01x^2$

$P' = 5 - .02x = 0$

$5 = .02x$

1,250 customers give max profit.

8. The Dobbs Hotel will provide a dinner party for a minimum of 100 couples at \$50 per couple. If more than 100 couples attend, the hotel will refund every couple \$.25 for every couple over 100. How many couples will maximize the hotel's revenue?

$P = (100+x)(50-.25x)$ $P' = 25 - .50x = 0$ 150 couples

$P = 5000 + 50x - 25x - .25x^2$ $25 = .5x$

$P = 5000 + 25x - .25x^2$ $50 = x$

9. A poster is to contain 50 square inches of printed matter with margins of 4" each at the top and bottom and 2" at each side. Find the *overall* dimensions if you want a minimum total area.

$xy = 50$ $x = 50/y$ $A = 50 + 8y + \frac{200}{y} + 32$ $8y^2 = 200$ overall dimensions $9 \times 18$ in

$A = (x+8)(y+4)$ $A' = 8 - \frac{200}{y^2} = 0$ $y^2 = 25$

$= xy + 8y + 4x + 32$ $y = 5,\ x = 10$ printed area

10. An oil can is to be made in the form of a right circular cylinder to contain $16\pi$ cubic inches. What dimensions of the can will require the least amount of material, while meeting this requirement for volume. *(Recall that for a cylinder, $V = \pi r^2 h$, and area of a circle $A = \pi r^2$.)*

$\pi r^2 h = 16\pi$ $SA = 2\pi r h + 2\pi r^2$ $A' = -\frac{32\pi}{r^2} + 4\pi r = 0$ $r^3 = 8$

$r^2 h = 16$ $= \frac{32\pi}{r} + 2\pi r^2$ $4\pi r = \frac{32\pi}{r^2}$ $r = 2$

$h = 16/r^2$ $h = 16/4 = 4$

11. The yield of orange trees is reduced if they are planted too close together. If there are 30 trees per acre, each tree produces 400 oranges. For each additional tree in the acre, the yield is reduced by 7 oranges per tree. How many trees per acre yield the largest crop for farmer Boyles?

$Y = (30+t)(400-7t)$ $Y' = 190 - 14t = 0$ 43 or 44 trees

$Y = 12000 + 400t - 210t - 7t^2$ $14t = 190$

$190t$ $t = 13.5$

12. A rectangular box is to be made from a piece of cardboard 24 inches long and 9 inches wide by cutting out identical squares from the four corners and turning up the cardboard to form the sides. What size square should you cut off of each corner to maximize the volume of the box? What is this maximum volume?


$V = x(24-2x)(9-2x)$ $V = 216x - 66x^2 + 4x^3$ $18 - 11x + x^2 = 0$ cut off 2x2 in squares

$= (24x - 2x^2)(9-2x)$ $V' = 216 - 132x + 12x^2$ $(9-x)(2-x) = 0$ $V = 2(20)(5)$

$= 216x - 18x^2 - 48x^2 + 4x^3$ $= 108 - 66x + 6x^2$ $x = 9, 2$ $V = 200\ cm^3$

$54 - 33x + 3x^2$

13. Rancher Sellers has 80 ft. of fence with which he plans to enclose a rectangular pen along one side of his 100 ft. barn (the side along the barn needs no fence). What are the dimensions that would maximize the area? What is the maximum area? $A = xy = 80x - 2x^2$


$2x + y = 80$ $A' = 80 - 4x = 0$

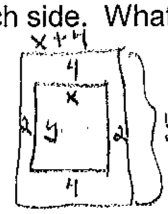
$y = 80 - 2x$ $80 = 4x$ max area $800\ ft^2$

$y = 40$ $20 = x$

14. A handbill is to contain 50 square inches, with 4 inch margins at the top and bottom and 2 inch margins on each side. What dimensions for the handbill would give the largest printed area?

printed area 1 by 2 in

handbill 5 by 10 in



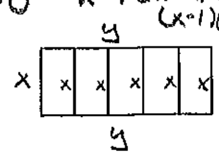

$(x+4)(y+8) = 50$ $A = xy$ $A' = \frac{(x+4)(18-16x) - (18x - 8x^2)(1)}{(x+4)^2} = 0$

$xy + 4y + 8x + 32 = 50$ $A = \frac{x(18-8x)}{x+4}$ $x = 1, -9$

$y(x+4) = 18 - 8x$ $18x + 72 - 16x^2 - 64x - 18x + 8x^2 = 0$ $y = \frac{18-8}{1+4} = \frac{10}{5} = 2$

$y = \frac{18-8x}{x+4}$ $A = \frac{18x - 8x^2}{x+4}$ $72 - 64x - 8x^2 = 0$ $x^2 + 8x - 9 = 0$ $(x-1)(x+9)$

15. A man with 300 m of fencing wishes to enclose a rectangular area and divide it into 5 pens with fences parallel to one side. (See figure at right)…
What dimensions would maximize the area?




$6x + 2y = 300$ $A = x(150 - 3x)$

$3x + y = 150$ $A = 150x - 3x^2$

$y = 150 - 3x$ $A' = 150 - 6x = 0$

$A = x \cdot y$ $150 = 6x$

$25 = x$ $y = 75$