Textbook p 248 #27, 28

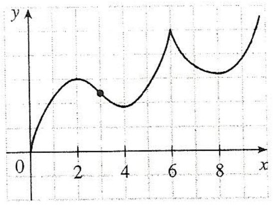
27. (a) f is increasing where f' is positive, that is, on $(0, 2)$, $(4, 6)$, and $(8, \infty)$; and decreasing where f' is negative, that is, on $(2, 4)$ and $(6, 8)$.

(b) f has local maxima where f' changes from positive to negative, at $x = 2$ and at $x = 6$, and local minima where f' changes from negative to positive, at $x = 4$ and at $x = 8$.

(c) f is concave upward (CU) where f' is increasing, that is, on $(3, 6)$ and $(6, \infty)$, and concave downward (CD) where f' is decreasing, that is, on $(0, 3)$.

(d) There is a point of inflection where f changes from being CD to being CU, that is, at $x = 3$.

(e)



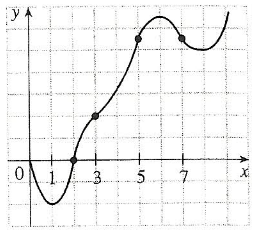
28. (a) f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.

(b) f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.

(c) f is concave upward where f' is increasing, that is, on $(0, 2)$, $(3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.

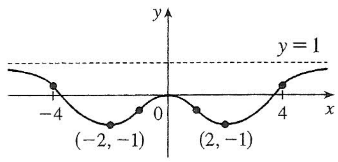
(d) There are points of inflection where f changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$ and $x = 7$.

(e)



Textbook p 262 #51

51. First we plot the points which are known to be on the graph: $(2, -1)$ and $(0, 0)$. We can also draw a short line segment of slope 0 at $x = 2$, since we are given that $f'(2) = 0$. Now we know that $f'(x) < 0$ (that is, the function is decreasing) on $(0, 2)$, and that $f''(x) < 0$ on $(0, 1)$ and $f''(x) > 0$ on $(1, 2)$. So we must join the points $(0, 0)$ and $(2, -1)$ in such a way that the curve is concave down on $(0, 1)$ and concave up on $(1, 2)$. The curve must be concave up and increasing on $(2, 4)$ and concave down and increasing toward $y = 1$ on $(4, \infty)$. Now we just need to reflect the curve in the y-axis, since we are given that f is an even function [the condition that $f(-x) = f(x)$ for all x].



Textbook p 309 #6, 13-19

6. $f(x) = \sin x + \cos^2 x$, $[0, \pi]$. $f'(x) = \cos x - 2\cos x \sin x = \cos x\,(1 - 2\sin x)$, so $f'(x) = 0$ for x in $(0, \pi)$ $\Leftrightarrow$ $\cos x = 0$ or $\sin x = \frac{1}{2}$ $\Leftrightarrow$ $x = \frac{\pi}{6}, \frac{\pi}{2}$, or $\frac{5\pi}{6}$. $f'(x) = \cos x - \sin 2x$ $\Rightarrow$ $f''(x) = -\sin x - 2\cos 2x$, so $f''\left(\frac{\pi}{6}\right) = -\frac{1}{2} - 2\left(\frac{1}{2}\right) = -\frac{3}{2}$, $f''\left(\frac{\pi}{2}\right) = -1 - 2(-1) = 1$, and $f''\left(\frac{5\pi}{6}\right) = -\frac{1}{2} - 2\left(\frac{1}{2}\right) = -\frac{3}{2}$. Thus, $f\left(\frac{\pi}{6}\right) = \frac{5}{4}$ and $f\left(\frac{5\pi}{6}\right) = \frac{5}{4}$ are local maxima and $f\left(\frac{\pi}{2}\right) = 1$ is a local minimum. $f(0) = 1$ and $f(\pi) = 1$, so f has its absolute minimum value of 1 at 0, $\frac{\pi}{2}$, and π. f attains its absolute maximum value of $\frac{5}{4}$ at $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

13. $f(0) = 0$, $f'(-2) = f'(1) = f'(9) = 0$, $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to 6} f(x) = -\infty$, $f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$, and $(9, \infty)$, $f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$, $f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$, $f''(x) < 0$ on $(0, 6)$ and $(6, 12)$