p369

(9) $\displaystyle\int_1^2 8x^3+3x^2\,dx = 2x^4+x^3\Big]_1^2$

$= 2(2)^4+2^3-(2\cdot 1^4+1^3)$

$= 32+8-(2+1) = \boxed{37}$

(11) $\displaystyle\int_0^1 1-x^9\,dx = x-\tfrac{1}{10}x^{10}\Big]_0^1$

$= 1-\tfrac{1}{10}(1)^{10}-(0-\tfrac{1}{10}(0)^{10})$

$= 1-\tfrac{1}{10}-0 = \boxed{\tfrac{9}{10}}$

(13) $\displaystyle\int_1^9 \frac{\sqrt{u}-2u^2}{u}\,du = \int_1^9 u^{-1/2}-2u\,du = 2u^{1/2}-u^2\Big]_1^9$

$= 2\sqrt{9}-9^2-(2\sqrt{1}-1^2) = 6-81-(2-1) = \boxed{-76}$

$\tfrac{1}{2}du$

(15) $\displaystyle\int_0^1 y\underbrace{(y^2+1)}_{u}^5\,dy$ $\quad u=y^2+1 \quad u(1)=1^2+1=2$

$\frac{du}{dy}=2y \quad u(0)=0^2+1=1$

$\displaystyle=\frac{1}{2}\int_{u(0)}^{u(1)} u^5\,du$ $\quad \tfrac{1}{2}du = y\,dy$

$= \tfrac{1}{2}\left(\tfrac{1}{6}u^6\right)\Big]_1^2 = \tfrac{1}{12}(2)^6-\tfrac{1}{12}(1)^6 = \tfrac{1}{12}(64)-\tfrac{1}{12} = \boxed{\tfrac{63}{12} = \tfrac{21}{4}}$

(17) $\displaystyle\int_1^5 \frac{dt}{(t-4)^2} = \int_1^5 (\underbrace{t-4}_{u})^{-2}\,\overset{du}{dt}$ $\quad u=t-4 \quad u(5)=5-4=1$

$du = 1\,dt \quad u(1)=1-4=-3$

$\displaystyle= \int_{u(1)}^{u(5)} u^{-2}\,du$

$= -1u^{-1}\Big]_{-3}^{1} = -1/1 - {}^{-1}/_{-3} = -1-\tfrac{1}{3} = -\tfrac{4}{3}$

Function discontinous at $t=4$

Integral doesn't exist!

$\frac{1}{3}du$

(19) $\int_0^1 v^2\cos(\underbrace{v^3}_{u})\,dv$ $\quad u=v^3 \quad u(1)=1^3=1$

$du=3v^2dv \quad u(0)=0^3=0$

$\frac{1}{3}\int_{u(0)}^{u(1)} \cos u\,du \quad \frac{1}{3}du=v^2dv$

$= \frac{1}{3}\sin u\Big]_0^1 = \frac{1}{3}\sin(1)-\frac{1}{3}\sin(0) = \boxed{\frac{1}{3}\sin(1)}$

$\frac{1}{2}du$

(21) $\int \frac{x+2}{\sqrt{\underbrace{x^2+4x}_{u}}}dx \quad u=x^2+4x$

$du=2x+4\,dx$

$=\frac{1}{2}\int \frac{1}{\sqrt{u}}du \quad du=2(x+2)dx$

$=\frac{1}{2}\int u^{-1/2}du \quad \frac{1}{2}du=x+2\,dx$

$=\frac{1}{2}\left(\frac{2}{1}u^{1/2}\right)+C = \boxed{(x^2+4x)^{1/2}+C}$

(23) $\int \underbrace{\sin(\pi t)}_{u}\underbrace{\cos(\pi t)\,dt}_{\frac{1}{\pi}du} \quad u=\sin(\pi t)$

$du=\pi\cos(\pi t)\,dt$ "chain rule"

$=\frac{1}{\pi}\int u\,du \quad \frac{1}{\pi}du=\cos(\pi t)dt$

$=\frac{1}{\pi}\left(\frac{1}{2}u^2\right)+C = \boxed{\frac{1}{2\pi}\sin^2(\pi t)+C}$

(25) $\int_0^{\pi/8} \sec(\underbrace{2\theta}_{u})\tan(\underbrace{2\theta}_{u})\underbrace{d\theta}_{\frac{1}{2}du} \quad u=2\theta \quad u\left(\frac{\pi}{8}\right)=2\left(\frac{\pi}{8}\right)=\pi/4$

$du=2\,d\theta \quad u(0)=2(0)=0$

$\frac{1}{2}\int_{u(0)}^{u(\pi/8)} \sec u\tan u\,du \quad \frac{1}{2}du=d\theta$

$=\frac{1}{2}\sec u\Big]_0^{\pi/4} = \frac{1}{2}\sec(\pi/4)-\frac{1}{2}\sec(0) = \frac{1}{2}(\sqrt{2})-\frac{1}{2}(1) = \boxed{\frac{\sqrt{2}}{2}-\frac{1}{2}}$

27) $\int_0^3 |x^2-4|\,dx = \int_0^2 -(x^2-4)\,dx + \int_2^3 (x^2-4)\,dx$

$= -\frac{x^3}{3} + 4x\Big]_0^2 + \frac{x^3}{3} - 4x\Big]_2^3$

$= \left[-\frac{2^3}{3} + 4(2) - \left(\frac{0^3}{3} - 4(0)\right)\right] + \left[\frac{3^3}{3} - 4(3) - \left(\frac{2^3}{3} - 4(2)\right)\right]$

$= \frac{-8}{3} + 8 + \frac{27}{3} - 12 - \frac{8}{3} + 8 = \frac{11}{3} + 4 = \boxed{\frac{23}{3}}$