p391 #1-23 odd

1. $\frac{dr}{ds} = .75r$

$\int \frac{dr}{r} = \int .75\,ds$

$\ln|r| = .75s + C$

$e \quad e$

$\boxed{r = Ae^{.75s}}$

3. $\frac{dy}{dx} = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + 2C$

$\boxed{y = \pm\sqrt{x^2+2C}}$

5. $\frac{dy}{dx} = \frac{x-1}{y^3}$

$y^3\,dy = x-1\,dx$

$\frac{y^4}{4} = \frac{x^2}{2} - x + C$

$y^4 = 2x^2 - 4x + 4C$

$\boxed{y = \pm\sqrt[4]{2x^2-4x+D}}$

7. $(2+x)y' = 3y$

$\frac{dy}{dx} = \frac{3y}{2+x}$

$\frac{dy}{3y} = \frac{dx}{2+x}$

$\frac{1}{3}\ln|y| = \ln|2+x| + C$

$\ln|y|^{1/3} = \ln|2+x| + C$

$e \quad e$

$y^{1/3} = A(2+x)$

$\boxed{y = B(2+x)^3}$

9. $yy' = 4\sin x$

$\frac{dy}{dx} = \frac{4\sin x}{y}$

$\int y\,dy = \int 4\sin x\,dx$

$\frac{y^2}{2} = -4\cos x + C$

$y^2 = -8\cos x + 2C$

$\boxed{y = \pm\sqrt{-8\cos x + D}}$

11. $\sqrt{1-4x^2}\,y' = x$

$\frac{dy}{dx} = \frac{x}{\sqrt{1-4x^2}}$

$\int dy = \int \frac{x\,dx}{\sqrt{1-4x^2}}$ $u = 1-4x^2$ $\frac{du}{dx} = -8x$

$y = -\frac{1}{8}\int u^{-1/2}\,du$

$y = -\frac{1}{8}(u)^{1/2}\cdot 2 + C$

$\boxed{y = -\frac{1}{4}\sqrt{1-4x^2} + C}$

13. $y\ln x - xy' = 0$

$y\ln x = x\frac{dy}{dx}$

$u = \ln x$

$\frac{du}{dx} = 1/x$

$\int \frac{\ln x}{x}\,dx = \int \frac{dy}{y}$

$\int u\,du = \ln|y| + C_y$

$\frac{1}{2}u^2 + C_u = \ln|y|$

$\frac{1}{2}(\ln x)^2 + C = \ln|y|$

$Ae^{\frac{1}{2}(\ln x)^2} = y$

15. $yy' - 2e^x = 0$ $\quad y(0) = 6$

$y\frac{dy}{dx} = 2e^x$

$\int y\,dy = \int 2e^x\,dx$

$\frac{y^2}{2} = 2e^x + C$

$y^2 = 4e^x + 2C$

$6^2 = 4e^0 + 2C$

$36 = 4 + 2C$

$32 = 2C$

$y^2 = 4e^x + 32$

$\boxed{y = +\sqrt{4e^x + 32}}$

17) $y(x+1) + y' = 0 \qquad (-2, 1)$

$$\frac{dy}{dx} = -y(x+1)$$

$$\frac{dy}{y} = -x - 1\,dx$$

$$\ln|y| = \frac{-x^2}{2} - x + C$$

$$e \qquad e$$

$$y = Ae^{-x^2/2 - x}$$

$$1 = Ae^{-(-2)^2/2-(-2)}$$

$$1 = Ae^0$$

$$1 = A$$

$$\boxed{y = e^{-x^2/2 - x}}$$

23) $dP - KPdt = 0$

$$dP = KPdt$$

$$\frac{dP}{P} = k\,dt$$

$$\ln|P| = kt + C$$

$$P = e^{kt+C} = Ae^{kt}$$

$$P_0 = Ae^{k(0)}$$

$$P_0 = A$$

$$\boxed{P = P_0 e^{kt}}$$

19) $y(1+x^2)y' - x(1+y^2) = 0 \qquad (0, \sqrt{3})$

$$y(1+x^2)\frac{dy}{dx} = x(1+y^2)$$

$u = 1+y^2$

$du = 2y\,dy$

$\frac{1}{2}du = y\,dy$

$$\frac{y\,dy}{1+y^2} = \frac{x\,dx}{1+x^2} \qquad w = 1+x^2$$

$$dw = 2x\,dx$$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\int \frac{dw}{w} \qquad \frac{1}{2}dw = x\,dx$$

$$\ln|u| = \ln|w| + C$$

$$\ln|1+y^2| = \ln|1+x^2| + C$$

$$e \qquad e$$

$$1+y^2 = A(1+x^2)$$

$$1+(\sqrt{3})^2 = A(1+0^2)$$

$$4 = A$$

$$1+y^2 = 4(1+x^2)$$

$$y^2 = 4(1+x^2) - 1$$

$$y = \sqrt{4(1+x^2)-1}$$

$$\boxed{y = \sqrt{4x^2+3}}$$

21) $\frac{du}{dv} = uv\sin v^2 \qquad (0,1)$

$$w = v^2$$

$$\frac{du}{u} = v\sin v^2\,dv \qquad dw = 2v\,dv$$

$$\ln|u| = \frac{1}{2}\int \sin w\,dw \qquad \frac{1}{2}dw = v\,du$$

$$\ln|u| = -\frac{1}{2}\cos w + C$$

$$\ln|u| = -\frac{1}{2}\cos(v^2) + C$$

$$u = Ae^{-\frac{1}{2}\cos(v^2)}$$

$$1 = Ae^{-1/2\cos 0}$$

$$1 = Ae^{-1/2}$$

$$e^{1/2} = A$$

$$u = e^{1/2}\cdot e^{-1/2\cos(v^2)}$$

$$u = e^{[1/2 - 1/2\cos(v^2)]}$$