p567 # 21-32

21) Integral Test

$\int_1^\infty \frac{1}{2n-1}\,dn = \lim_{a\to\infty} \frac{1}{2}\ln|2n-1|\Big]_1^a = \infty$ ∴ Series diverges

$u = 2n-1$
$du = 2dn$ $\frac{1}{2}\int\frac{1}{u}$

22) Integral Test

$\int_2^\infty \frac{1}{n\sqrt{n^2-1}}\,dn = \lim_{a\to\infty} \sec^{-1} n\Big]_2^a = \frac{\pi}{2} - \frac{\pi}{3}$ ∴ series converges

23) p-series

$p = 5/4 > 1$ ∴ series converges

24) p-series

$p = .95 \leq 1$ ∴ Series diverges

25) geometric series

$r = \frac{2}{3} < 1$ ∴ series converges

26) n^{th} term test

$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$ ∴ series diverges

27) geometric series

$r = 1.075 \geq 1$ ∴ series diverges

28) $\sum_{n=1}^{\infty} \frac{1}{n^2} - \sum_{n=1}^{\infty} \frac{1}{n^3}$ both p-series ∴ series converges

$p = 2 > 1$ $p = 3 > 1$ both converge

29) n^{th} term test

$$y = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = 1^\infty$$

$$\ln y = \lim_{n\to\infty} n \ln\left(1+\frac{1}{n}\right) = \infty \cdot 0$$

$$\ln y = \lim_{n\to\infty} \frac{\ln\left(\frac{n+1}{n}\right)}{1/n} = \frac{0}{0}$$

$$\ln y = \lim_{n\to\infty} \frac{\frac{1}{\frac{n+1}{n}}\left(\frac{n\cdot 1-(n+1)1}{n^2}\right)}{-1/n^2}$$

$$\ln y = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)\left(\frac{-1}{\cancel{n^2}}\right)\left(\frac{-\cancel{n^2}}{1}\right) = 1$$

$e \qquad e$

$$y = e \neq 0$$

$\therefore$ series diverges

30) n^{th} term test

$$\lim_{n\to\infty} \ln n = \infty \neq 0 \quad \therefore \text{ series diverges}$$

31) integral test

$$\int_2^\infty \frac{1}{n(\ln n)^3}\,dn = \lim_{a\to\infty} \frac{\ln n^{-2}}{-2}\Big]_2^a = \lim_{a\to\infty} \frac{1}{-2(\ln a)^2} - \frac{1}{-2[\ln 2]^2} = 0 + \frac{1}{2(\ln 2)^2}$$

$\therefore$ series converges

$u = \ln n \quad \int u^{-3}$

$du = 1/n \, dn$

32) integral test

$$\int_2^\infty \ln n(n^{-3})\,dn = \lim_{a\to\infty} \frac{\ln a}{-2n^2}\Big|_2^a - \int_2^a \frac{1}{-2n^3}\,dn$$

$u = \ln n \quad dv = n^{-3}\,dn$

$du = 1/n \quad v = \frac{n^{-2}}{-2}$

$$= \lim_{a\to\infty} \frac{\ln n}{-2n^2} + \frac{1}{2}\frac{n^{-2}}{-2}\Big|_2^a$$

$$= \lim_{a\to\infty} \frac{\ln a}{-2a^2} - \frac{1}{4a^2} - \left(\frac{\ln 2}{-2\cdot 2^2} - \frac{1}{4\cdot 2^2}\right)$$

$$\frac{\infty}{\infty} - 0 + \frac{\ln 2}{8} + \frac{1}{16}$$

$$\overset{H}{=} \lim_{a\to\infty} \frac{1/a}{-4a} - 0 + \frac{\ln 2}{8} + \frac{1}{16}$$

$$= 0 - 0 + \frac{\ln 2}{8} + \frac{1}{16}$$

$\therefore$ Series converges