

p586 # 1-13 odd, 21-25 odd, 31-48 all

1. $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!} \right| = \infty > 1 \therefore \text{diverges}$ by ratio test

3. $\lim_{n \rightarrow \infty} \left| \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} \right| = 0 < 1 \therefore \text{converges}$ by root test

5. $\lim_{n \rightarrow \infty} \left| \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} \right| = \frac{1}{2} < 1 \therefore \text{converges}$

7. $\lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{(n+1)^2} \cdot \frac{n^2}{2^n} \right| = 2 > 1 \therefore \text{diverges}$

9. $\lim_{n \rightarrow \infty} \left| \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right| = 0 < 1 \therefore \text{converges}$

11. $\lim_{n \rightarrow \infty} \left| \frac{(n+1)!}{(n+1)3^{n+1}} \cdot \frac{n3^n}{n!} \right| = \infty > 1 \text{ diverges}$

13. $\lim_{n \rightarrow \infty} \left| \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n} \right| = 0 < 1 \text{ converges}$

$y = \lim_{n \rightarrow \infty} \sqrt[n]{n}$
 $\ln y = \lim_{n \rightarrow \infty} \frac{1}{n} \ln n$
 $\ln y = \lim_{n \rightarrow \infty} \frac{1/n}{1} = 0$
 $y = 1$

21) $\lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n}$
 $= \frac{1}{2} < 1$
 converges

23) $\lim_{n \rightarrow \infty} \sqrt[n]{\left|\frac{(-1)^n}{(\ln n)^n}\right|}$
 $= \lim_{n \rightarrow \infty} \frac{1}{|\ln n|} = 0 < 1$
 $\therefore \text{converges}$

25) $\lim_{n \rightarrow \infty} \sqrt[n]{(2\sqrt{n} + 1)^n}$
 $\lim_{n \rightarrow \infty} 2\sqrt{n} + 1$
 $= 2(1) + 1 = 3 > 1$

$$a_{n+1} < a_n$$

31. $\lim_{n \rightarrow \infty} \frac{5}{n} = 0$
 \therefore converge
 by AST

32. p-series
 (Harmonic $p=1$)
 \therefore diverges

33. p-series
 $p = 3/2 > 1$
 \therefore converges

34) geo series
 $r = \frac{\pi}{4} = r < 1$
 \therefore converges

35. $\lim_{n \rightarrow \infty} \frac{2n}{n+1} = 2 \neq 0$
 \therefore diverges
 by n^{th} term test

36. diverges
 LCT
 compare to $\frac{1}{n}$
 $\lim = \frac{1}{2} \geq 0$

37. diverges
 Ratio Test
 $\lim = \frac{3}{2} > 1$

38. converges by LCT
 compare to $\frac{1}{n^{3/2}}$
 $\lim = 1/3 \neq 0$

39. converge by LCT
 compare to $\frac{1}{2^n}$
 $\lim = 1/0 \neq 0$

40. $\lim_{n \rightarrow \infty} \frac{2^n}{4n^2 - 1} = \infty$
 diverge n^{th} term

$$a_{n+1} < a_n$$

41. $\lim_{n \rightarrow \infty} \ln\left(\frac{n+2}{n}\right) = 0$
 converge by AST

42. diverge LCT
 compare to $\frac{1}{\sqrt{n}}$
 $\lim = 1 \neq 0$

43. $\frac{\cos n}{2^n} < \frac{1}{2^n}$
 converge by DCT

$$a_{n+1} < a_n$$

44. $\lim_{n \rightarrow \infty} \frac{1}{n \ln n} = 0$
 \therefore converge by AST

45. converge
 ratio test
 $\lim = 0 < 1$

46. $\frac{\ln(n)}{n^2} < \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}$
 converge DCT

47. converge
 ratio test
 $\lim = 0 < 1$

48. converge
 ratio test
 $\lim = 0 < 1$