

3.4 Position, Velocity, and Acceleration

1. A balloonist drops a sandbag from a balloon 160 feet above the ground. After t seconds, the sandbag is $160 - 16t^2$ feet above the ground.
 a) Find the velocity of the sandbag at $t=1$.
 b) With what velocity does the sandbag hit the ground?

$v(3.16227766) = -101.193 \text{ ft/sec}$

$v(t) = -32t$
 $v(1) = -32 \text{ ft/sec}$
 $0 = 160 - 16t^2$
 $16t^2 = 160$
 $t^2 = 10$
 $t = 3.16227766$

2. A projectile is fired directly upward from the ground with an initial velocity of 112 ft/sec. Its distance above the ground after t seconds is $112t - 16t^2$ feet.

- a) Find the velocity of the projectile at $t = 2, t=3$, and $t=4$.
 b) When does the projectile hit the ground?
 c) Find the velocity at the instant it hits the ground.

$v(7) = -112 \text{ ft/sec}$

$v(t) = 112 - 32t$
 $v(2) = +48 \text{ ft/sec}$
 $v(3) = +16 \text{ ft/sec}$
 $v(4) = -16 \text{ ft/sec}$
 $t(112 - 16t) = 0$
 $112 = 16t$
 $t = 7 \text{ sec}$

3. A projectile is shot upward from the surface of the earth with an initial velocity of 384 ft/sec. If $s(t) = 384t - 16t^2$, what is its velocity after 5 seconds? After 10 seconds?

$v(t) = 384 - 32t$
 $v(5) = 224 \text{ ft/sec}$
 $v(10) = 64 \text{ ft/sec}$

4. A pebble is dropped from a height of 600 feet. If $s(t) = 600 - 16t^2$, find the pebble's velocity when it hits the ground.

$0 = 600 - 16t^2$
 $16t^2 = 600$
 $t^2 = 37.5$
 $t = 6.123724357$
 $v(t) = -32t$
 $v(6.123724357) = -195.959 \text{ ft/sec}$

5. A ball is thrown straight down from the top of a 220 foot building with an initial velocity of -22 ft/sec. If $s(t) = 220 - 22t - 16t^2$,

- a) What is the velocity after 3 seconds?
 b) What is its velocity after falling 108 feet?

$s(t) =$
height

$v(t) = -22 - 32t$
 $v(3) = -22 - 32(3) = -118 \text{ ft/sec}$
 $220 - 108 = 112 \text{ ft high}$
 $v(2) = -56 \text{ ft/sec}$

6. A ball is dropped from a height of 100 feet and $s(t) = 100 - 16t^2$ ft/sec. One second later another ball is dropped from a height of 75 feet and $s(t) = 75 - 16t^2$ ft/sec. Which one hits the ground first?

$16t^2 = 100$
 $t = 2.5$
 $16t^2 = 75$
 $t = 2.16 + 1 \text{ sec} = 3.16 \text{ sec}$
 $\therefore 1^{\text{st}} \text{ ball hits first}$

7. An object is dropped from a balloon which is stationary at 1600 ft. Express its height above the ground as a function of t . (Hint: Look at the pattern of the equations in 1-6). How long does it take the object to reach the ground?

$s(t) = 1600 - 16t^2$
 $0 = 1600 - 16t^2$
 $16t^2 = 1600$
 $t^2 = 100$
 $t = 10 \text{ sec}$

8. A ball is thrown vertically upward from the ground with an initial velocity of 60 ft/sec. How high will the ball go?

$s(t) = 60t - 16t^2$
 $v(t) = 60 - 32t$
 $0 = 60 - 32t$
 $1.875 = t$
 $v(1.875) = 56.25 \text{ ft}$
 $\max(1.875, 56.25)$

9. A ball is thrown upward with an initial velocity of 64 ft/sec from a height of 80 ft.
 a) Find the position function for this motion.
 b) When will the ball hit the ground?

a) $s(t) = 80 + 64t - 16t^2$
 $80 + 64t - 16t^2 = 0$
 $5 + 4t - t^2 = 0$
 $(5 - t)(1 + t) = 0$
 $t = 5, -1$

max is turning pt
 up \rightarrow down
 $+ \rightarrow -$
 \therefore when $v(t) = 0$

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10) a) $v(t) = 6t^2 - 30t + 36$ $a(t) = 12t - 30$

b) $a(1) = -18 \text{ m/sec}^2$

c) $v(t) = 6t^2 - 30t + 36 = 0$ $a(2) = -6 \text{ m/s}^2$
 $6(t-2)(t-3) = 0$

$t = 2 \text{ or } 3$ $a(3) = 6 \text{ m/s}^2$

11) $v(t) = 4t^3 - 12t^2$ $a(t) = 12t^2 - 24t$

$12t^2 - 24t = 0$

$12t(t-2) = 0$

$t = 0 \text{ or } 2 \text{ sec}$

$s(0) = 2 \text{ m}$ $s(2) = -14 \text{ m}$

$v(0) = 0 \text{ m/s}$ $v(2) = -16 \text{ m/s}$

15. (a) Since the slope of the tangent at $t = 0$ is 0, the car's initial velocity was 0.

(b) The slope of the tangent is greater at C than at B , so the car was going faster at C .

(c) Near A , the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up. Near B , the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near C is the one at C , so at C the car had just finished speeding up, and was about to start slowing down.

(d) Between D and E , the slope of the tangent is 0, so the car did not move during that time.

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