Properties of Integrals

Quick Review 5.3

In Exercises 1–10, find dy/dx.

1. $y = -\cos x$
2. $y = \sin x$
3. $y = \ln(\sec x)$
4. $y = \ln(\sin x)$
5. $y = \ln(\sec x + \tan x)$
6. $y = x \ln x - x$
7. $y = \dfrac{x^{n+1}}{n+1} \quad (n \neq -1)$
8. $y = \dfrac{1}{2^x + 1}$
9. $y = xe^x$
10. $y = \tan^{-1} x$

Section 5.3 Exercises

The exercises in this section are designed to reinforce your understanding of the definite integral from the algebraic and geometric points of view. For this reason, you should not use the numerical integration capability of your calculator (NINT) except perhaps to support an answer.

1. Suppose that f and g are continuous functions and that

$$\int_1^2 f(x)\,dx = -4, \quad \int_1^5 f(x)\,dx = 6, \quad \int_1^5 g(x)\,dx = 8.$$

Use the rules in Table 5.3 to find each integral.

(a) $\displaystyle\int_2^2 g(x)\,dx$ (b) $\displaystyle\int_5^1 g(x)\,dx$

(c) $\displaystyle\int_1^2 3f(x)\,dx$ (d) $\displaystyle\int_2^5 f(x)\,dx$

(e) $\displaystyle\int_1^5 [f(x) - g(x)]\,dx$ (f) $\displaystyle\int_1^5 [4f(x) - g(x)]\,dx$

2. Suppose that f and h are continuous functions and that

$$\int_1^9 f(x)\,dx = -1, \quad \int_7^9 f(x)\,dx = 5, \quad \int_7^9 h(x)\,dx = 4.$$

Use the rules in Table 5.3 to find each integral.

(a) $\displaystyle\int_1^9 -2f(x)\,dx$ (b) $\displaystyle\int_7^9 [f(x) + h(x)]\,dx$

(c) $\displaystyle\int_7^9 [2f(x) - 3h(x)]\,dx$ (d) $\displaystyle\int_9^1 f(x)\,dx$

(e) $\displaystyle\int_1^7 f(x)\,dx$ (f) $\displaystyle\int_9^7 [h(x) - f(x)]\,dx$

3. Suppose that $\int_1^2 f(x)\,dx = 5$. Find each integral.

(a) $\displaystyle\int_1^2 f(u)\,du$ (b) $\displaystyle\int_1^2 \sqrt{3} f(z)\,dz$

(c) $\displaystyle\int_2^1 f(t)\,dt$ (d) $\displaystyle\int_1^2 [-f(x)]\,dx$

4. Suppose that $\int_{-3}^0 g(t)\,dt = \sqrt{2}$. Find each integral.

(a) $\displaystyle\int_0^{-3} g(t)\,dt$ (b) $\displaystyle\int_{-3}^0 g(u)\,du$

(c) $\displaystyle\int_{-3}^0 [-g(x)]\,dx$ (d) $\displaystyle\int_{-3}^0 \frac{g(r)}{\sqrt{2}}\,dr$

5. Suppose that f is continuous and that

$$\int_0^3 f(z)\,dz = 3 \quad \text{and} \quad \int_0^4 f(z)\,dz = 7.$$

Find each integral.

(a) $\displaystyle\int_3^4 f(z)\,dz$ (b) $\displaystyle\int_4^3 f(t)\,dt$

6. Suppose that h is continuous and that

$$\int_{-1}^1 h(r)\,dr = 0 \quad \text{and} \quad \int_{-1}^3 h(r)\,dr = 6.$$

Find each integral.

(a) $\displaystyle\int_1^3 h(r)\,dr$ (b) $\displaystyle -\int_3^1 h(u)\,du$