Key

Review

A function f is continuous over the interval described.

x	-3	-3 < x < 0	0	0 < x < 3	3	3 < x < 7	7	7 < x < 10	10
f	0	neg	-4	neg	0	pos	6	pos	0
f'	0	neg	DNE	pos	0	pos	DNE	Neg	0
f''	neg	neg	DNE	neg	0	pos	DNE	pos	pos

a. Find the x-coordinates of all relative extrema on the domain [-3, 10]. Classify them as relative max or relative mins. Justify your answer.

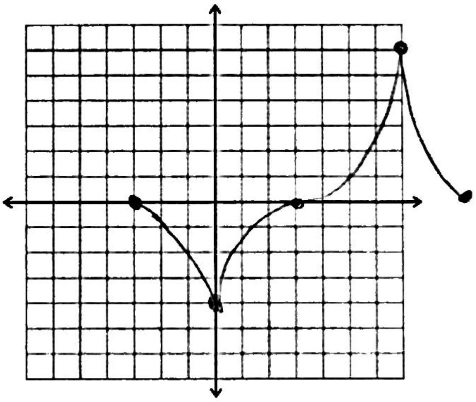
X = 0 is a rel. min. f' changes from neg to pos indicating f is changing from dec to inc.

X = 7 is a rel max. f' changes from pos to neg indicating f is changing from inc to dec.

b. Find the x-coordinates of any point of inflection on the domain [-3, 10]. Justify your answer.

X = 3 is an inflection pt. f'' changes from neg to pos indicating f(x) changes from concave down to concave up.

c. Sketch a possible graph of f(x).



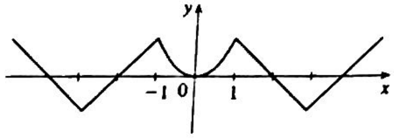
Textbook p 239 #11, 13

11. $f(x) = 3x^2 + 2x + 5$, $[-1, 1]$. f is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$ since polynomials are continuous and differentiable on $\mathbb{R}$. $f'(c) = \dfrac{f(b) - f(a)}{b - a} \iff 6c + 2 = \dfrac{f(1) - f(-1)}{1 - (-1)} = \dfrac{10 - 6}{2} = 2 \iff 6c = 0 \iff c = 0$, which is in $(-1, 1)$.

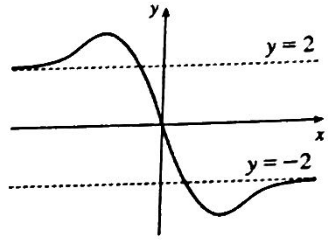
13. $f(x) = \sqrt[3]{x}$, $[0, 1]$. f is continuous on $\mathbb{R}$ and differentiable on $(-\infty, 0) \cup (0, \infty)$, so f is continuous on $[0, 1]$ and differentiable on $(0, 1)$. $f'(c) = \dfrac{f(b) - f(a)}{b - a} \iff \dfrac{1}{3c^{2/3}} = \dfrac{f(1) - f(0)}{1 - 0} \iff \dfrac{1}{3c^{2/3}} = \dfrac{1 - 0}{1} \iff$ $3c^{2/3} = 1 \iff c^{2/3} = \frac{1}{3} \iff c^2 = \left(\frac{1}{3}\right)^3 = \frac{1}{27} \iff c = \pm\sqrt{\frac{1}{27}} = \pm\frac{\sqrt{3}}{9}$, but only $\frac{\sqrt{3}}{9}$ is in $(0, 1)$.

p309

14. For $0 < x < 1$, $f'(x) = 2x$, so $f(x) = x^2 + C$. Since $f(0) = 0$, $f(x) = x^2$ on $[0, 1]$. For $1 < x < 3$, $f'(x) = -1$, so $f(x) = -x + D$. $1 = f(1) = -1 + D \;\Rightarrow\; D = 2$, so $f(x) = 2 - x$. For $x > 3$, $f'(x) = 1$, so $f(x) = x + E$. $-1 = f(3) = 3 + E \;\Rightarrow\; E = -4$, so $f(x) = x - 4$. Since f is even, its graph is symmetric about the y-axis.



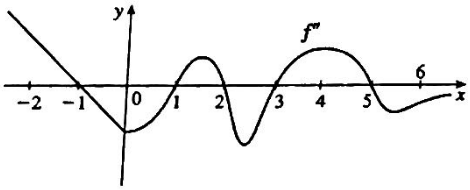
15. f is odd, $f'(x) < 0$ for $0 < x < 2$, $f'(x) > 0$ for $x > 2$, $f''(x) > 0$ for $0 < x < 3$, $f''(x) < 0$ for $x > 3$, $\lim_{x\to\infty} f(x) = -2$



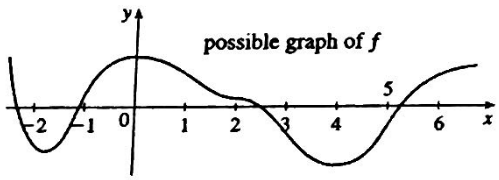
16. (a) Using the Test for Monotonic Functions we know that f is increasing on $(-2, 0)$ and $(4, \infty)$ because $f' > 0$ on $(-2, 0)$ and $(4, \infty)$, and that f is decreasing on $(-\infty, -2)$ and $(0, 4)$ because $f' < 0$ on $(-\infty, -2)$ and $(0, 4)$.

(b) Using the First Derivative Test, we know that f has a local maximum at $x = 0$ because f' changes from positive to negative at $x = 0$, and that f has a local minimum at $x = 4$ because f' changes from negative to positive at $x = 4$.

(c)

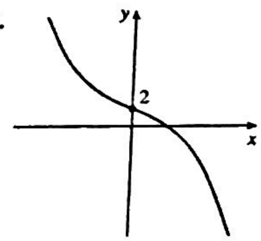


(d)



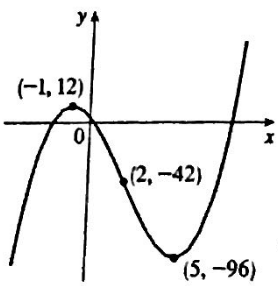
17. $y = f(x) = 2 - 2x - x^3$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 2$. The x-intercept (approximately 0.770917) can be found using Newton's Method. **C.** No symmetry **D.** No asymptote
E. $f'(x) = -2 - 3x^2 = -(3x^2 + 2) < 0$, so f is decreasing on $\mathbb{R}$.
F. No extreme value **G.** $f''(x) = -6x < 0$ on $(0, \infty)$ and $f''(x) > 0$ on $(-\infty, 0)$, so f is CD on $(0, \infty)$ and CU on $(-\infty, 0)$. There is an IP at $(0, 2)$.

H.



18. $y = f(x) = x^3 - 6x^2 - 15x + 4$ **A.** $D = \mathbb{R}$
B. y-intercept: $f(0) = 4$; x-intercepts: $f(x) = 0 \;\Rightarrow\; x \approx -2.09, 0.24, 7.85$ **C.** No symmetry **D.** No asymptote
E. $f'(x) = 3x^2 - 12x - 15 = 3(x^2 - 4x - 5) = 3(x + 1)(x - 5)$, so f is increasing on $(-\infty, -1)$, decreasing on $(-1, 5)$, and increasing on $(5, \infty)$. **F.** Local maximum value $f(-1) = 12$, local minimum value $f(5) = -96$. **G.** $f''(x) = 6x - 12 = 6(x - 2)$, so f is CD on $(-\infty, 2)$ and CU on $(2, \infty)$. There is an IP at $(2, -42)$.

H.



19. $y = f(x) = x^4 - 3x^3 + 3x^2 - x = x(x - 1)^3$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \;\Leftrightarrow\; x = 0$ or $x = 1$ **C.** No symmetry **D.** f is a polynomial function and hence, it has no asymptote.
E. $f'(x) = 4x^3 - 9x^2 + 6x - 1$. Since the sum of the coefficients is 0, 1 is a root of f', so $f'(x) = (x - 1)(4x^2 - 5x + 1) = (x - 1)^2(4x - 1)$. $f'(x) < 0 \;\Rightarrow\; x < \frac{1}{4}$, so f is decreasing on $(-\infty, \frac{1}{4})$ and f is increasing on $(\frac{1}{4}, \infty)$. **F.** $f'(x)$ does not change sign at $x = 1$, so there is not a local extremum there. $f(\frac{1}{4}) = -\frac{27}{256}$ is a local minimum value. **G.** $f''(x) = 12x^2 - 18x + 6 = 6(2x - 1)(x - 1)$.
$f''(x) = 0 \;\Leftrightarrow\; x = \frac{1}{2}$ or 1. $f''(x) < 0 \;\Leftrightarrow\; \frac{1}{2} < x < 1 \;\Rightarrow\; f$ is CD on $(\frac{1}{2}, 1)$ and CU on $(-\infty, \frac{1}{2})$ and $(1, \infty)$. There are inflection points at $(\frac{1}{2}, -\frac{1}{16})$ and $(1, 0)$.

H.