Optimization + RR Review

1) 

$V = \frac{4}{3}\pi r^3 \quad \frac{dV}{dt} = 4.5 \quad r = 2 \quad \frac{dr}{dt} = ?$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4.5 = 4\pi(2)^2 \frac{dr}{dt}$$

$$\boxed{.090 \text{ ft/min} = \frac{dr}{dt}}$$

2) 

$\frac{dV}{dt} = 10 \quad d = 3h \quad h = 15 \quad \frac{dh}{dt} = ?$

$2r = 3h$

$r = \frac{3h}{2}$

$V = \frac{1}{3}\pi r^2 h \quad V = \frac{1}{3}\pi\left(\frac{3h}{2}\right)^2 h$

$V = \frac{1}{3}\pi \frac{9}{4} h^3$

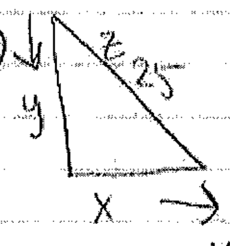
$V = \frac{3\pi}{4} h^3$

$\frac{dV}{dt} = \frac{9\pi}{4} h^2 \frac{dh}{dt}$

$10 = \frac{9\pi}{4}(15)^2 \frac{dh}{dt}$

$10 = \frac{2025}{4}\pi \frac{dh}{dt}$

$$\boxed{\frac{dh}{dt} = \frac{40}{2025\pi} \approx .006 \text{ ft/min}}$$

3) 

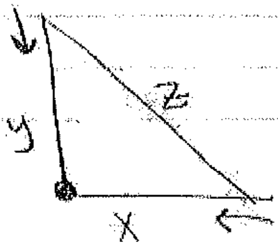
$x = 7 \quad \frac{dx}{dt} = 2 \quad z = 25 \quad \frac{dz}{dt} = 0$

$y = 24 \quad \frac{dy}{dt} = ?$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt} \quad 7(2) + 24\frac{dy}{dt} = 0 \quad \frac{dy}{dt} = \frac{-7}{12} \text{ ft/s}$

(4)



$x = 200 \quad \frac{dx}{dt} = -600$

$y = 150 \quad \frac{dy}{dt} = -450$

$z = 250 \quad \frac{dz}{dt} = ?$

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$200(-600) + 150(-450) = 250\frac{dz}{dt}$

$\boxed{-750\text{mph} = \frac{dz}{dt}}$

5) $P = (1000 + x)(.85 - .01x)$

$P = 850 + .85x - 10x - .01x^2$

$P = 850 - 9.15x - .01x^2$

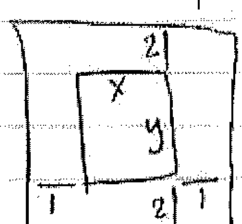
$P' = -9.15 - .02x = 0$

$-.02x = 9.15$

$x = -457.5$

$\therefore$ Sell when cow is 1542.5 lbs

(6)



$\frac{30}{\sqrt{15}} + 4$

$y + 4$

$30 = xy$

$\frac{30}{x} = y$

$A = (x+2)(y+4)$

$= xy + 4x + 2y + 8$

$= 30 + 4x + 2\left(\frac{30}{x}\right) + 8$

$x + 2$

$\sqrt{15} + 2 \quad \frac{30}{\sqrt{15}} + 4$

$\sqrt{15} + 2$

$\boxed{11.746 \text{ in by } 5.873 \text{ in}}$

$A' = 4 - \frac{60}{x^2} = 0$

$4 = \frac{60}{x^2}$

$\sqrt{15} = x$

t	2	
v	$-$	$+$
a		$+$

c) $(2, \infty)$

7) a) $v(t) = 3t^2 - 12$
$a(t) = 6t$

b) down $(0, 2)$
up $(2, \infty)$

d) $3 \to -13 \to -6$
16 7
23 total

$L(x) = 63.2(x-10) + 332.7$

$D(11) \approx 395.9$

$D'(10) = \frac{332.7 - 269.5}{10 - 9} = 63.2$

$D'(2) = 13.3$
$\frac{10 + 16.6}{2}$

8) $D'(2) = \frac{29.9 - 3.3}{3 - 1} = \frac{26.6}{2} = 13.3$ ft/sec

$D'(2)$ is the rate the distance traveled is changing at 2 seconds in ft. per second.

or

$D'(2)$: the rate of change of the distance traveled at 2 seconds is 13.3 ft/sec.
(velocity at 2 seconds in ft/sec)

9) $f(x) = \sqrt{x^2 + 9}$ $x = -4$
$f(-4) = 5$
$f'(x) = \frac{1}{2}(x^2+9)^{-1/2} \cdot 2x$
$f'(-4) = \frac{-4}{\sqrt{(-4)^2+9}} = \frac{-4}{5}$

$\boxed{y = \frac{-4}{5}(x+4) + 5}$

approx $\sqrt{25.81}$. $\boxed{\approx 5.08035}$
$25.81 = x^2 + 9$
$= x^2$
$\pm\sqrt{16.81} = x = \pm 4.1$
Take -4.1 as x because it is closest to -4. (the value used for tangent line.)

$y = \frac{-4}{5}(-4.1 + 4) + 5 = 5.08$
$= \frac{4}{5}\left(-\frac{1}{10}\right) + 5$

(10) $f(x) = \sqrt[3]{27-x}$ approx $\sqrt[3]{26}$

$x = ?$ what # can I easily plug into the equation to evaluate and still be close to $\sqrt[3]{26}$? $x = 0$

$\sqrt[3]{27} = 3$

$f(0) = 3$

$\sqrt[3]{27}$ is near $\sqrt[3]{26}$

$f'(x) = \frac{1}{3}(27-x)^{-2/3} \cdot -1$

$= \frac{-1}{3(\sqrt[3]{27-x})^2}$

$$\boxed{y = \frac{-1}{27}(x-0)+3}$$

$f'(0) = \frac{-1}{3(3)^2}$

approx $\sqrt[3]{26} = \sqrt[3]{27-x}$

$26 = 27 - x$

$1 = x$

$= \frac{-1}{27}$

$y = \frac{-1}{27}(1) + 3$

$\sqrt[3]{26}$ $\boxed{\approx 2.963}$